RELIABILITY- BASED STABILITY ANALYSIS OF SLOPE

Tarini Jahnavi

Department of Civil Engineering
National Institute of Technology Rourkela

RELIABILITY- BASED STABILITY ANALYSIS OF SLOPE

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by

Tarini Jahnavi

(214CE1081)

Under the guidance of

Dr. Shantanu Patra

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Department of Civil Engineering

National Institute of Technology Rourkela

Department of Civil Engineering
National Institute of Technology Rourkela

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Certificate of Examination

Roll Number: *214CE1081*

Name: *Tarini Jahnavi*

Title of Thesis: Reliability- Based Stability Analysis of Slope

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Department of Civil Engineering
National Institute of Technology Rourkela

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This is to certify that the work presented in the thesis entitled **Reliability- Based Stability Analysis of Slope** *submitted by* **Tarini Jahnavi (Roll No: 214CE1081)** *is a record of original research carried out by her under my supervision and guidance in partial fulfilment of the requirements of the degree of* **Master of Technology** *in Department of* **Civil Engineering** *with specialization in* **"Geotechnical Engineering"** *at* **National Institute of Technology**, **Rourkela.** *Neither this thesis nor any part of it has been submitted earlier for any degree or diploma to any institute or university in India or abroad.*

Prof. Shantanu Patra

Declaration of Originality

I *Tarini Jahnavi*, *Roll No: 214CE1081* hereby declare that this thesis entitled *Reliability-Based Stability Analysis of Slope* presents my original work carried out as a student of NIT Rourkela and to the best of my knowledge, contains no material previously published or written by another person, nor any material presented by me for the award of any degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the thesis. Works of other authors cited in this thesis have been duly acknowledged under the sections “Reference”. I have also submitted my original research records to the scrutiny committee for evaluation of my thesis. I am fully aware that in case of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present thesis.

May 25, 2016 *Tarini Jahnavi*

NIT Rourkela

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(T.Jahnavi)

Abstract

In Geotechnical engineering, the design and construction is done based on the Factor of Safety obtained from the deterministic approach. This Factor of safety doesn't take into account the source and amount of uncertainty associated with the soil properties. Therefore, reliability based approach for the stability analysis has to be done to consider these uncertainties. In the present study, reliability-based stability analysis of slope has been made for using Finite Element Method, Upper bound Limit Analysis and Analytical method given by Low (1989). The commercially available software PLAXIS 2D-V9.02 is used for Finite Element Method and LimitState:GEO for Limit Analysis. The limit state function is developed using response surface methods. Full factorial design is used for development of response surface models. In this study, reliability analysis is done using first order reliability method. The need for reliability analysis and the corresponding reliability index and factor of safety is discussed. The study is validated by analysing a case study of James Bay dykes. Parametric study has been done varying the soil and slope properties and modification has been made in the equation given by Low's equation of Factor of Safety.

Keywords: deterministic; reliability; finite element method; limit analysis; reliability index; probability of failure.

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Chapter 1

Introduction

In geotechnical engineering analysis and design various sources of uncertainties are encountered and well recognized. Traditionally, a deterministic approach is used for slope stability analysis. However, the determination of variables such as soil strength parameters, pore pressure and other pertinent properties involves uncertainties, which cannot be handled in the traditional deterministic methods. It is, therefore, highly desirable to apply a reliability based analytical/numerical methodology for stability analysis of dams taking into account these uncertainties.

Several features usually contribute to such uncertainties, like: (1) those associated with inherent randomness of natural processes; (2) Model uncertainty reflecting the inability of the simulation model, design technique or empirical formula to represent the system's true physical behaviour, such as calculating the safety factor of slopes using limiting equilibrium methods of slices; (3) Model parameter uncertainties resulting from inability to quantify accurately the model input parameters and (4) Data uncertainties including (a) measurement errors, (b) data inconsistency and non-homogeneity and (c) data handling (Malkawi 2000) .

In slope stability computations, various sources of uncertainties are encountered, such as geological details missed in the exploration program, estimation of soil properties that are difficult to quantify, i.e. the spatial variability in the field cannot be reproduced accurately, fluctuation in pore water pressure, testing errors and many other relevant factors.

In a deterministic analysis, the factor of safety (F) is defined as the ratio of resisting to driving forces on a potential sliding surface. The slope is considered safe only if the calculated safety factor clearly exceeds unity. Whereas, in a probabilistic framework the factor of safety is expressed in terms of its mean value as well as its variance. Reliability analysis is therefore used to assess uncertainties in engineering variables such as the factor of safety of slope stability. The reliability index (β) is often used to express the degree of uncertainty in the calculated factor of safety. Such uncertainty is usually assessed by different approaches such as the first-order second-moment method, point estimate method, and Monte Carlo simulation method.

Chapter 2

Literature Review

Whitman (1984) stated that risk and reliability analyses are theoretically very useful in the course of the initial stages of a project in making decision to continue or not and in assisting the establishment of design criteria. He suggested that reliability can be used as guide for the selection of safety factor reliable with the degree of safety in case if the problem is understood with enough data but with no standard factor of safety. In case of different slopes, reliability theory can be used to improve consistency in the safety. It is important to differentiate the spatial variability effect and systematic errors effect. In economical point of view, optimization techniques are helpful for making the choice of safety factor. Even if there is any doubt in the computed numerical results, systematic formulation of the reliability problem can very helpful in understanding of it. It is not possible to calculate the actual risk precisely only by analysis in cases where the risk has to be little. Still the outline of risk evaluation can guide the subjective evaluations.

Low (1989) proposed an easy and expedient semi-analytical procedure for embankment's factor of safety built on soft clay. He developed Stability numbers N_1, N_2 respectively for normalized foundation strength, normalized embankment strength. The safety factor is calculated as the sum of the two products of stability numbers with their corresponding normalized strength. A circular potential slip surface was assumed. The significance of the foundation and embankment strengths can be individually compared during the calculation. The computation can also consider the cases with soft clays of varied undrained shear strength with depth and for c-ϕembankment soils.

Christian et al. (1994) described how laboratory and field data are used for deriving the probabilistic values of soil parameters which are applicable in stability analysis. He explored the first order second moment approach and applied to the embankment dams design. The comparative effect of uncertainties of different parameters is also considered. The uncertainty in the soil properties comprises of scatter and systematic error. The first covers random measurement error and real spatial variability and the latter contains both statistical uncertainty and the effects of bias. For establishing the factor of safety for the design which represent the reliable risks for different modes of failure, reliability analysis is very useful. Reliability index quantifies stability better than factor of safety as it describes safety by the standard deviations separating F from its standard failure value, 1.0. Thus, making it an implicit approach.

Greco (1996) presented an efficient Monte-Carlo method for analysis of slope stability to locate the critical slip surface. The process consists of number of stages where a suitable technique is generated for every new slip surface by a repetitive procedure, depending on generating random numbers. The framework of method was very simple, programmed easily, integrated, and modified for particular necessities. It is sturdy enough for layered soils which are feeble, thin, inclined layers and critical problem for the search of slip surface. The suggested method provides results as good as the best methods of nonlinear programming.

Low et al. (1997) proposed a practical method to calculate the Hasofer-Lind second moment reliability index β using spreadsheets. The proposed technique is dependent on the perception of ellipsoid which is tangential to surface of failure in original space of random variables. By forming the quadratic arrangement of tilted ellipsoid in the spreadsheet, correlation is considered. Using the nonnormal and its corresponding normal relationship, the nonnormals are dealt. Spreadsheet's optimization tool performs automatically the numerical partial differentiation and iterative searching. Because of its relative easiness and perceptiveness, the suggested method can be a striking alternative to the established mathematical tools which require closed-form partial derivatives and transformed space.

Low et al. (1997) proposed a powerful spreadsheet technique to reduce the iterations required for the first order reliability method (FORM) and Janbu's generalized procedure of slices (GPS) applicable for slope stability analysis with slip surface which is non-circular. In this method, the principal concepts were made clearer to user, intuitions were obtained. The spreadsheet's optimization tool automatically calculated the optimization partial derivatives. The search for the non-circular critical slip surface for both deterministic and probabilistic is automatic inspite of the point that the expressions for safety factor and the performance functions were implicit. This technique is also applicable for other generalized limit equilibrium methods of slopes.

Liang et al. (1999) established reliability and probability theories to calculate reliability index along with its corresponding probability of failure of multi-layered slopes and embankment dams. A computerized program named RESLOP was introduced which was confirmed by failure case of Congress Street open cut. The stability of King Talal embankment dam was studied using this approach. The uncertainties in the properties were incorporated in the analytical process by using the reliability analysis approach. The obtained reliability index comprises of additional data when compared to the deterministic safety factor which helps in finding the slip surface that is most likely for the slope rather than the surface with the minimum factor of safety alone.

Malkawi et al. (2000) presented a procedure for slope stability analysis in probabilistic approach using Monte Carlo simulation method and first-order second-moment method. A comparison was done with results obtained from these methods with four familiar techniques of slope stability analysis. These are Ordinary method, simplified Janbu's method, Spencer's method and simplified Bishop's method. In case of homogeneous slope, the reliability index β for Ordinary and Bishop Methods were in good agreement but for Janbu and Spencer methods there exists variation between MCSM and FOSM. In case of non-homogeneous slope β is near for Ordinary, Janbu and Bishop Methods but different for Spencer method.

Ramly et al. (2002) introduced a spreadsheet technique for analysis of probabilistic slope stability which is based on MC simulation by using easily obtainable softwares, @Risk and MS Excel 97. The study takes into account, spatial variability of input parameters and statistical uncertainty because of inadequate data and biases in the correlations and empirical factors used. The methodology was applied to probabilistic slope stability analysis of the dykes of the James Bay hydroelectric project and for the comparison of the outcomes was done with the first-order second-moment method. Reliability index and probability of unsatisfactory performance were calculated by combining the factor of safety and uncertainty involved in it. The decision making process can be improved by linking the conventional slope stability analysis and probabilistic based approach for slope engineering practice.

Griffiths et al. (2004) have studied the failure probability with simple and advanced probabilistic analysis methods for a cohesive slope. The simple tool considered strength of complete slope to be only random input variable without local averaging and spatial correlation such that it falls lower than a critical value depending on a lognormal pdf. The

Random FEM based on elastoplastic nonlinear analyses using MCS is capable of considering both local averaging and spatial correlation and allows the mechanism of failure to pass through weakest plane in soil. Thus, obtained values of failure probability using simplified analysis based on probabilistic approach are unconservative.

Low (2005) illustrated a practical design process based on reliability for retaining walls using FORM and the Hasofer–Lind index. Random variables which are both normally and non-normally correlated are taken into consideration. Based on Low & Tang work, the probabilistic spreadsheet-based approach attains the similar outcome compared with FORM and the Hasofer–Lind method, but utilises an instinctive perspective of expanding dispersion ellipsoid which significantly make interpretations and computations simpler. Sensitivity analysis for the random variables in reliability analysis was done. The comparison of probabilities of failure obtained from reliability index and Monte Carlo simulations was done.

Foye et al. (2006) presented a structure of Load & Resistance factor design (LRFD) factors based on reliability based design approach and orderly method for selecting the Probability Density Functions. The uncertainty in the design parameters is dependent on the transformation and material uncertainties. There is an increase in composite variable uncertainty with increase in transformation and material uncertainties. For clay and sand, the uncertainties in bearing capacity equations are systematically analysed. Numerical integration of the fundamental equations has been used to define the PDFs that are necessary to perform a reliability analysis of the bearing capacity of footings.

Xu et al. (2006) proposed a reliability based approach using response surface method for combining probabilistic stability analysis and FEM in case of embankments. It was presented that the deterministic based analyses of model has great effect on the results of reliability based analyses for embankments. Any assumption made in analytical model based on deterministic approach, for example using a non-rigorous method or circular slip surface leads to an inexact valuation of reliability index. Particularly, the analysis considering circular slip surface will overestimate the reliability index if the shape of slip circle highly influences the value of factor of safety. The suggested reliability analysis technique via RSM is mainly useful for combining several deterministic stability analyses like limit equilibrium methods, finite-element method, and FORM to form a probabilistic stability analysis.

Massih et al. (2008) presented a method based on reliability for the design and analysis of shallow strip footing with a vertical load both considering and without considering loading of pseudostatic seismicity. The study of ultimate limit state is done considering only the punching failure mode. They have shown that there is an increase in foundation reliability when the correlation is negative in between the shear strength parameters of soil. The factors which influence the probability of failure are the coefficient of variation of the horizontal seismic coefficient and soil angle of internal friction.

Cho (2010) proposed a statistical process in which uncertainties are taken into consideration for the problems of slope stability. The process explores from Limit Equilibrium Method of slices which is deterministic analysis to the approach based on probability which considers the spatial variation and uncertainties of soil parameters. The failure probability is higher when it is considered overall rather than for assumed critical surface. The reason is that the critical slip surface found by searching through algorithm gives more or less the same safety factor when linked to the one by considering assumed critical surface for every variable field. For slopes made of undrained saturated clay, this condition is satisfied because of its spatial uncertainty in shear strength but for the cohesive friction soils, there is a good correlation for slip surfaces using limit state equations.

Wang et al. (2011) developed a Monte Carlo simulation (MCS)-based reliability analysis approach for slope stability problems and utilizes Subset simulation, an advanced MCS method for improving efficiency and resolution of the MCS at relatively small probability levels. To explore the effect of spatial variability of the soil properties and critical slip surface spreadsheet package was used. By assuming perfect correlation, spatial variability of soil properties is ignored which results in the overestimation of variance of the factor of safety (FS). This may lead to either conservative or unconservative estimation of the probability of failure.

Low (2014) described an instinctive ellipsoidal perception combined with three spreadsheet-automated controlled optimization FORM processes and a SORM method. These methods were compared using some examples of a rock slope, a confined soil element, and embankment on soft ground having spatially auto correlated undrained shear strength in the foundation. Re-formulated Spencer method is considered for the performance function. The critical slip surface which is noncircular is searched based on reliability approach. The proposed methods were also compared with MCS.

Ray et al. (2015) explored an analytical study of a cantilever sheet pile wall taking the influence of uncertainties in soil properties into consideration. A factor named probabilistic risk factor (R_f) was proposed by combing the Probability of failure (P_f) of sheet pile wall with the sensitivity (S) of random input variables on the mode of failure. The value of P_f was attained using Finite Element model through Response Surface. F-test analysis was performed to do the Sensitivity analysis of every random variable. The water table positions and the cohesion parameter of foundation soil are the parameters which mostly affect the stability of the pile wall. For varying properties of soil, variations in height of cantilever sheet pile wall and different positions of water table, the suggested risk factor based method is beneficial.

Chapter 3

Objectives and Methodology

3.1 Objectives

The objective of this study is to perform reliability analysis of slope using Finite Element Method, Limit Analysis Method and Analytical method from which Reliability index and Probability of Failure are obtained. To compare the three analyses with the results from literature.

3.2 Methodology

For the slope reliability problem, the analysis procedure can be summarized by the following steps:

1. The parameters that are to be considered as random variables are identified such that they have noteworthy effect on the embankment stability. Generally the soil parameters like cohesion (c), angle of internal friction (ϕ) and unit weight (γ), thickness of the layers of foundation soil, and pore water pressure (u') in case of effective conditions are chosen as the random variables.

2. Each input variable is sampled for two values, ($\mu_i+m\sigma_i$) and ($\mu_i-m\sigma_i$). μ_i is the mean of the random variable, σ_i is its standard deviation and m is any arbitrary number. The value of 'm' is chosen anything. Full Factorial Design is used for developing the experimental design of a stability problem. According to this design, if a problem contains 'n' random variables, 2^n number of sampling points are required to form the response surface for the performance function.

3. The factors of safety corresponding to the design sample points are calculated using Finite Element method, Limit State Method and Analytical Method given by Low, 1989 separately. The commercially available software PLAXIS is used for Finite Element analysis and LimitState:GEO is used for Limit state analysis.

4. The linear response surface representing the performance function for the embankment is constructed from the input variable and their corresponding calculated FOS. After obtaining an approximate performance function, using First Order Reliability Method, the Hasofer–Lind reliability index, β_{HL} is found out by minimizing it in MS Excel Solver and probability of failure P_f is calculated from the reliability index.

3.2.1 Finite Element Method (FEM)

Finite element method (FEM) is a numerical method for solving a differential or integral equation. It has been applied to a number of physical problems, where the governing differential equations are available. The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution.

PLAXIS 2D-V9.02:

PLAXIS is a finite element software for soil and rock that has been used by geotechnical engineers and researchers for more than two decades. It is specifically used for stability and deformation analysis in geotechnical applications. The software was first developed by the Technical University of Delft in 1987 to analyse soft soils of the low lands of Holland (Brinkgreve and Vermeer, 2001). The software then was extended to cover all aspects and applications of geotechnical engineering simulation using a user-friendly interface with the power of finite element. The first version of PLAXIS was commercially available in 1998.

The program uses a convenient graphical user interface that enables users to quickly generate a geometry model and finite element mesh based on a representative vertical cross section of the situation hand. The problem can be modelled either by a plane strain or an axisymmetric model. The program has advantageous feature that enable user to choose different soil model which is dependent on mechanical deformation behaviour of soil for the simulation. The models include Mohr-Coulomb, joint rock, hardening soil, soft soil and modified cam-clay model. Standard boundary conditions are automatically generated by the program. Finite element mesh is easily generated from the input 2D geometry model. Automatic mesh generator with the bandwidth optimizer for the finite-element discretization allows generating finite element mesh (of thousands of element) with option for mesh refinement.

The calculation program is the part of the whole simulation where the analysis of the generated model is performed. The procedure is through definition/calculation of the staged construction step (steps that the model is build up). The program offers three types of calculation for the user in each construction phase: plastic, consolidation and safety. Before final calculation (whole problem), the user can choose specific points that load-displacement curves, stress path and stress strain curves can be generate for those points in output part. The program produces outputs of: deformed mesh of the model, different types of deformation and strain, effective and total stress. Complex finite element models can be generated easily through the program due to relatively simple graphical input procedure and the enhanced output facilities make available a detailed presentation of computational results.

3.2.2 Limit Analysis Method

The limit analysis of structures is a method to determine the maximum load parameter or increasing load parameter that a perfect elastic-plastic construction is able to take. Limit Analysis procedures are rigorously based upon the theorems of plasticity. Compared to the incremental analysis, the efficiency of the limit analysis is achieved by considering the final state, state of failure, without paying attention to what was happening with the construction and load from the moment when one section of the structure was completely plasticized until the failure. Limit analysis methods are based on the theorem of plastic failure of an ideal elasto-plastic body. These theorems are known as static (lower) and kinematic (upper) theorems of the marginal analysis of structures.

LimitState:GEO

LimitState:GEO is a general purpose software program which was designed to rapidly analyse the ultimate limit state or collapse state for a wide variety of geotechnical problems. The software can be used to model 2D problems of any geometry specified by the user including slopes, retaining walls, foundations, pipelines, tunnels, anchors etc. and any combination of these. It directly determines the ultimate limit state (ULS) using the computational limit analysis technique Discontinuity Layout Optimization (DLO).

The procedure was developed at the University of Sheffield and was first described in a paper published in the Proceedings of the Royal Society (Smith & Gilbert 2007). DLO can be used to identify critical translational sliding block failure mechanisms, output in a form which will be familiar to most geotechnical engineers. However while traditional methods can typically only work with mechanisms involving a few sliding blocks, DLO has no such limitations. It can identify the critical translational failure mechanism for any geotechnical stability problem, to a user specified geometrical resolution.

Discontinuity Layout Optimization (DLO) involves the use of rigorous mathematical optimization techniques to identify a critical layout of lines of discontinuity which form at failure. These lines of discontinuity are typically slip-lines in planar geotechnical stability problems and define the boundaries between the moving rigid blocks of material which make up the mechanism of collapse. Associated with this mechanism is a collapse load factor, which will be an upper bound on the exact load factor according to formal plasticity theory. Thus in essence the procedure replicates and automates the traditional upper bound hand limit analysis procedure.
The different model available are Mohr-Coulomb, Tension and/or compression cut off, rigid and engineered element. In addition, material models may be combined to generate more complex yield surfaces. The presence of water can be represented by a Water Table which affects the whole model and/or Water Regimes which can be assigned on a per-zone basis.

LimitState:GEO solves problems in terms of Adequacy factor. The Adequacy factor is defined as the factor by which material strengths decreased, or, specified loads must be increased in order for the system under consideration to reach a collapse state. There are thus two types of Adequacy factor used in the software: Adequacy factor on load and Adequacy factor on strength. LimitState:GEO is designed to work closely with the Eurocode 7 approach to Ultimate Limit State design. In Eurocode 7 Design Approach 1, partial factors are pre-applied to loads (as multipliers) and/or material properties (as divisors) prior to analysis. Assessment of safety is then undertaken by testing whether in the subsequent analysis, the available resistance to collapse exceeds the actions causing collapse. The setting of Partial Factor values is carried out using the Scenario Manager.

3.2.3 Response Surface Method (RSM)

Response surface method (RSM) is a collection of mathematical and statistical techniques for empirical model building. The method was introduced by Box and Wilson in 1951. By careful design of experiments, the objective is to optimize a response (output variable) which is influenced by several independent variables (input variables). An experiment is a series of tests, called runs, in which changes are made in the input variables in order to identify the reasons for changes in the output response. It is a powerful approach for carrying out reliability analysis for complicate engineering with implicit limit state functions.

In most *RSM* problems, the true response function *f* is unknown. In order to develop a proper approximation for *f*, the experiment is usually started with a low-order polynomial in some small region (Bradley, 2007). If the response can be defined by a linear function of independent variables, then the approximating function is a first-order model. A first-order model with 2 independent variables can be expressed as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$$

Here ε includes both experimental error and the effects of any uncontrolled factors in the experiment. The terms $\beta_1 x_1$ and $\beta_2 x_2$ are main effects and the term $\beta_{12} x_1 x_2$ is a two-way interaction effect. A designed experiment would systematically manipulate x_1 and x_2 while measuring y, with the objective of accurately estimating β_0, β_1, β_2, and β_{12}.

If there is a curvature in the response surface, then a higher degree polynomial should be used. The approximating function with 2 variables is called a second-order model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_{11}^2 + \beta_{22} x_{22}^2 + \beta_{12} x_1 x_2 + \varepsilon$$

The linear and non=linear response surfaces are shown in Figure 3.1, 3.2 respectively.

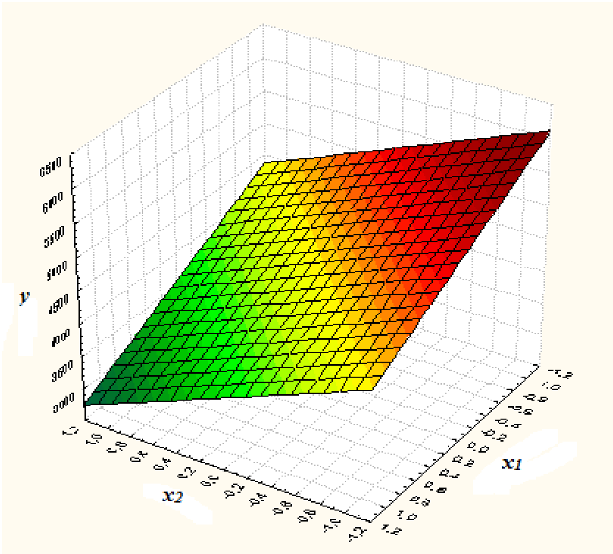


Figure 3.1: Linear Response surface

Polynomial models are generalized to any number of predictor variables x_i (i = 1, N) as follows:

$$y = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \sum_{j=1}^{k} \beta_{jj} x_j^2 + \sum_{i<j} \sum_{j=1}^{k} \beta_{ij} x_i x_j$$

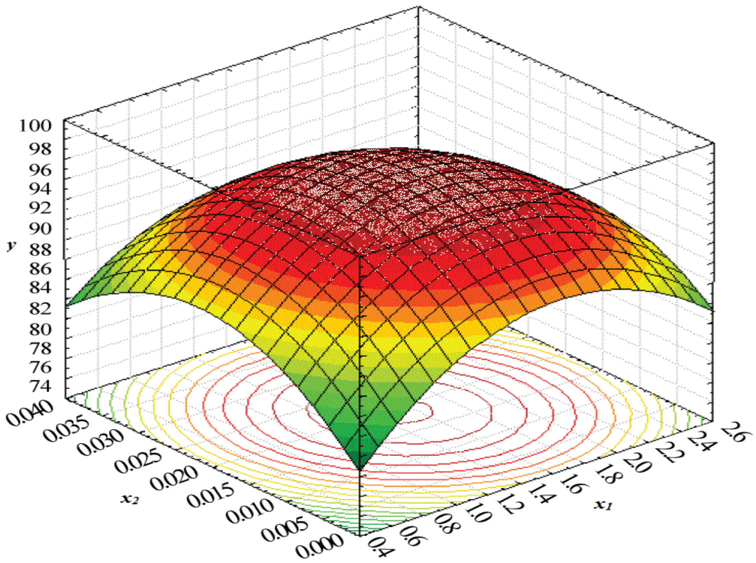


Figure 3.2: Nonlinear Response surface

In general all *RSM* problems use either one or the mixture of the both of these models. In each model, the levels of each factor are independent of the levels of other factors. In order to get the most efficient result in the approximation of polynomials the proper experimental design must be used to collect data. Once the data are collected, the Method of Least Square is used to estimate the parameters in the polynomials. The response surface analysis is performed by using the fitted surface.

Design of experiments:

An important aspect of RSM is the design of experiments (Box and Draper, 1987), usually abbreviated as DoE. The objective of DoE is the selection of the points where the response should be evaluated. A factorial experiment is an experimental strategy in which design variables are varied together, instead of one at a time. The lower and upper bounds of each of *N* design variables in the optimization problem needs to be defined. The allowable range is then discretized at different levels. If each of the variables is defined at only the lower and upper bounds (two levels), the experimental design is called 2^N full factorial. Similarly, if the midpoints are included, the design is called 3^N full factorial. The 2^N design is the basic building block. So this is used to create other response surface designs. A 2^N design is useful at the start of a response surface study.

Matlab code for design of experiments for 2 level factorial design is:

```
dFF2 = ff2n(n)
```

dFF2 is R-by-C, where R is the number of treatments in the full-factorial design. Each row of dFF2 corresponds to a single treatment. Each column contains the settings for a single factor, with values of 0 and 1 for the two levels. The binary set don’t have any meaning and simply considered as design set. If the number of parameters involved in the design is 3, then the design can be generated in Matlab as follows. These binary set don’t have any meaning and simply considered as design set.

```
>> dFF2 = ff2n(3)
dFF2 =
0 0 0
0 0 1
0 1 0
```

0 1 1

1 0 0

1 0 1

1 1 0

1 1 1

0 and 1 are then estimated as μ+mσ and μ-mσ. μ is the mean of the variable, σ is standard deviation of the corresponding variable and m is an arbitrary value. The decoded design sets are used to conduct experiments and output response is obtained. Using the set of input-output parameters linear or nonlinear regression model is developed using MS Excel.

3.2.4 RELIABILITY ANALYSIS

***Reliability*:**

The most common practical tools to evaluate the uncertainty in the output are

1. First-Order Second Moment (FOSM) approach,
2. First-Order Reliability Method (FORM),
3. Second- order reliability method (SORM),
4. Monte Carlo simulation techniques and
5. Event tree analysis.

Terminology:

Mean:

It is average or expected value of data set. It measures the central tendency of data. It is known as first central moment. For a random value X, the mean μ_x or the expected value E[X] is defined by,

$$E[X] = \mu_x = \frac{1}{n}\sum x_i$$

Variance:

It is the measure of spread in the data about the mean or average of the sample. It is known as second central moment. It is calculated using

$$Var[X] = \frac{1}{n-1}\sum (x_i - \mu_x)^2$$

Standard Deviation: (σ_x)

The Standard Deviation is related to the Variance by,

$$\sigma_x = \sqrt{Var[X]}$$

Coefficient of Variation: (CoV)

It is the measure of dispersion of data. If the CoV is higher than dispersion will be higher about its mean.

$$CoV[X] = \frac{\sigma_x}{\mu_x} * 100\%$$

Covariance:

Covariance indicates the degree of linear relationship between two random variables (x, y).

$$cov[X, Y] = \frac{1}{n-1} \sum (x_i - \mu_x)(y_i - \mu_y)$$

Correlation coefficient (ρ_{xy}):

It is a non dimensional parameter. It is obtained by dividing the covariance of two random variables cov[X, Y] with the product of standard deviation of individual variables (σ_x , σ_y)

$$\rho_{xy} = \frac{cov[X, Y]}{\sigma_x \sigma_y}$$

The correlation coefficient varies between -1 to +1. If the ρ_{xy} is high then the two random variables have high correlation. These are mostly linear dependent variable.

***Probability Density Function* (PDF):**

The PDF defines the distribution of the random variable and can take many shapes, but the most common in geotechnical applications are the normal and lognormal.

The PDF for the normal distribution with a mean, μ, and standard deviation, σ, is defined by

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

This distribution is symmetric about the mean, and the random variable can take on values between $-\infty$ to $+\infty$.

The PDF for the normal distribution with a mean, μ_N, and standard deviation, σ_N, is defined by

$$f(y;\mu_N,\sigma_N) = \frac{1}{\sigma_N x\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(x)-\mu_N}{\sigma_N}\right)^2\right]$$

Where $\sigma_N = \sigma_{\ln X} = \sqrt{\ln\left(1+{V_X}^2\right)}$

$\mu_N = \mu_{\ln X} = \ln(\mu_X) - \frac{1}{2}{\sigma_N}^2$

The lognormal Distribution ranges between zero and infinity.

First-Order Reliability Method (FORM):

Hasofer & Lind (1974) proposed an invariant definition for the reliability index. The approach is referred to as the first-order reliability method (FORM). The FORM employs a linearization of each limit state function at the design point, which is the point on the limit state surface nearest to the origin in the standard normal space. The distance from the origin to the limit state surface in the standard normal space represents the reliability index β.The starting point for FORM is the definition of the performance function G(X), where X is the vector of basic random variables. If the joint probability density function of all random variables $F_x(X)$ is known, then the probability of failure P_f is given by

$$P_f = P[G(U) < 0] = \int_L F_x(X)\,dX$$

Where, L is the domain of X where G(X) < 0.

In general**,** the above integral cannot be solved analytically. In the FORM approximation, the vector of random variables X is transformed to the standard normal space U, where U is a vector of independent Gaussian variables with zero mean and unit standard deviation, and where G(U) is a linear function. An illustration of the design point and graphical representation of β is given in Figure 3.3.

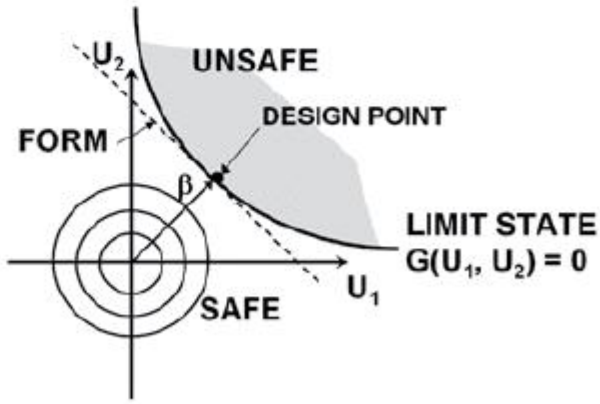


Figure 3.3: The FORM approximation and definition of β and design point

The probability of failure (Pf) can be estimated from the reliability index β, the distance between the origin and the hyperplane G(U) = 0 using the established equation

$P_f = 1- \Phi(\beta) = \Phi(-\beta)$

where Φ is the cumulative distribution (CDF) of the standard normal variate. The relationship is exact when the limit state surface is planar and the parameters follow normal distributions, and approximate otherwise. The relationship between the reliability index and probability of failure defined by Equation is shown in Figure 3.4.

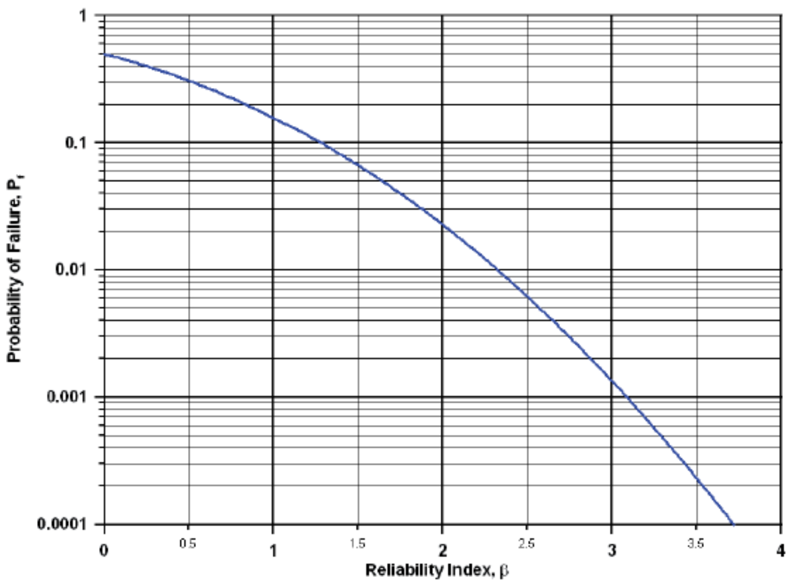


Figure 3.4: Relationship between reliability index β, and probability of failure Pf

Low (2003) presented a method for finding the reliability index in the original space. His approach is based on the matrix formulation of the Hasofer-Lind reliability index β x_i μ_i σ_i

$$\beta = min\sqrt{[\mathrm{X}-\mu]^T[C]^{-1}[\mathrm{X}-\mu]} \text{ for } \{X:G(X)=0\}$$

or, equivalently:

$$\beta = min\sqrt{\left[\frac{x_i-\mu_i}{\sigma_i}\right]^T [R]^{-1} \left[\frac{x_i-\mu_i}{\sigma_i}\right]} \text{ for } \{X:G(X)=0\}$$

Low and Tang (1997) used latter equation because the correlation matrix R is easier to set up, and conveys the correlation structure more explicitly than the covariance matrix C. The key advantage of this formulation is that it can be implemented using built-in functions in EXCEL without programming. By using Microsoft Excel's built-in Solver optimization tool to minimize β with the constraint that G[U]=0, and by automatically changing the values of the random variables, x_i. This spreadsheet-based technique and its intuitive ellipsoidal perspective in the original space of the random variables are referred to as the ellipsoid method.

CHAPTER 4

Stability Analysis of a hypothetical slope

A slope with frictional fill on purely cohesive soil is considered for this study. The cross section of the embankment is shown in Figure 4.1. The embankment is 6m high with a slope of 20º and the depth of foundation layer is 12m. The angle of internal friction of the fill material is 30º and the undrained shear strength of foundation soil is 30 kPa.

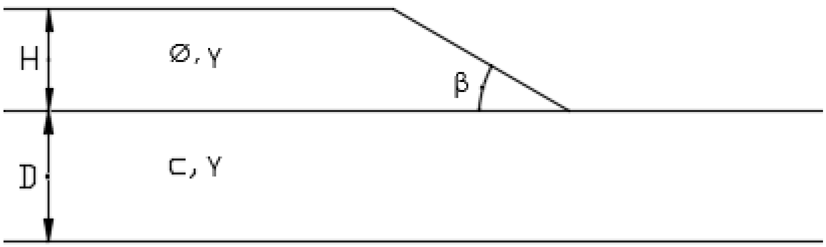


Figure 4.1: Cross Section of the embankment

4.1 Finite Element method

4.1.1 Deterministic Analysis of slope

The slope is modelled using the available software PLAXIS 2D-V9.02 as shown in Figure 4.2 Full fixity is considered at the bottom of the foundation soil and horizontal fixity at the sides of the model. The soil is represented using simple elastic, perfectly plastic Mohr- Coulomb model. Table 4.1shows the soil parameters of the embankment. The soil is modelled using 15-noded triangular elements with the 12-point integration rule. The Young's Modulus, E of embankment material is taken as 100 MPa and that for foundation material as 30 MPa. The Poisson's ratio, υ of both the materials is considered 0.30.

Table 4.1: Soil parameters of the embankment

	γ (kN/m3)	S_u (kPa)	ϕ (deg)
Embankment fill	20	0	30
Foundation soil	20	30	0

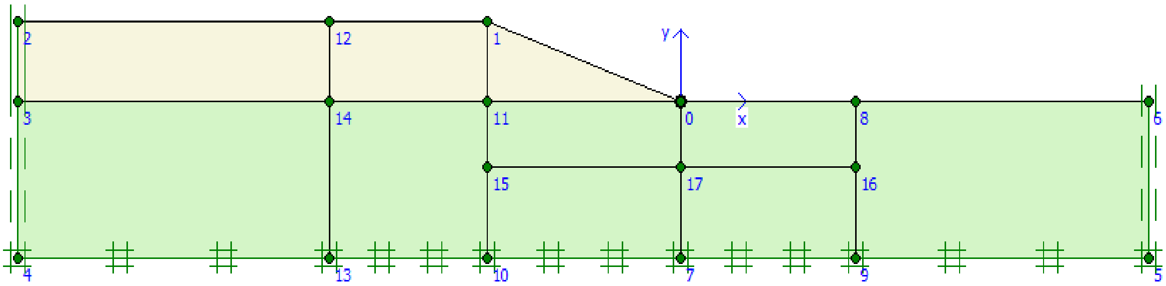


Figure 4.2: PLAXIS model of Slope

The mesh is generated using Fine coarseness globally. Clusters are formed in the critical areas of the slope and foundation. These clusters are refined further to increase the no. of elements using cluster refinement. The lines forming the boundaries of the clusters are also refined using the Line refinement. The refinement around the crest and toe nodes of the slope is done using Point refinement. The meshing details are shown in Figure 4.3.

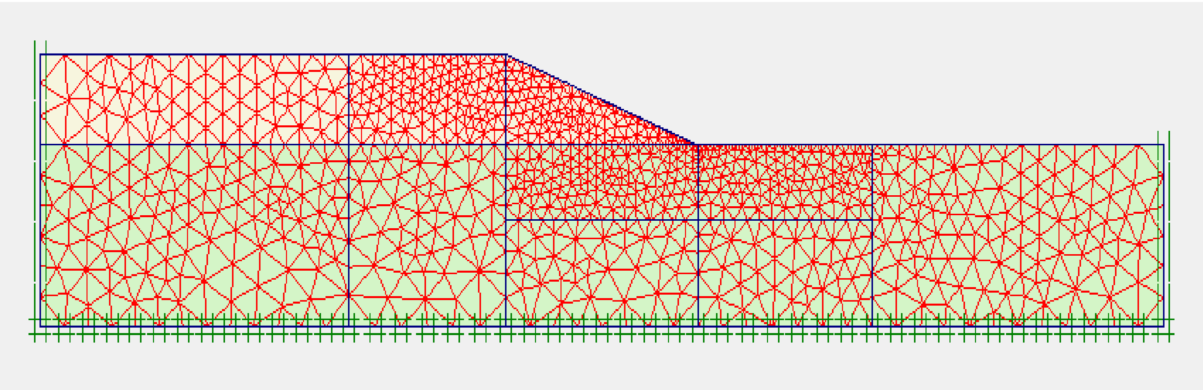


Figure 4.3: Meshing details of the slope

Phi/c reduction method in PLAXIS is used for calculating the Factor of Safety. It is represented as sum of incremental multiplier, ΣMsf and is defined as the ratio of the available shear strength to the shear strength at failure. Figure 4.4 shows the Deformed Mesh of the slope and the critical slip surface (Figure 4.5) is represented by Shear shadings of incremental strains.

$$\text{FS} = \frac{\text{available shear strength}}{\text{shear strength at failure}}$$

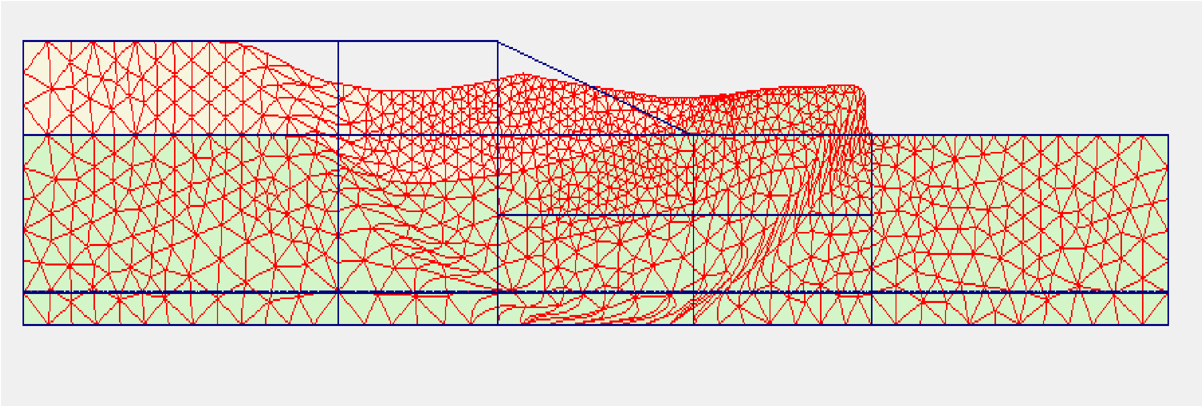


Figure 4.4: Deformed mesh of the slope

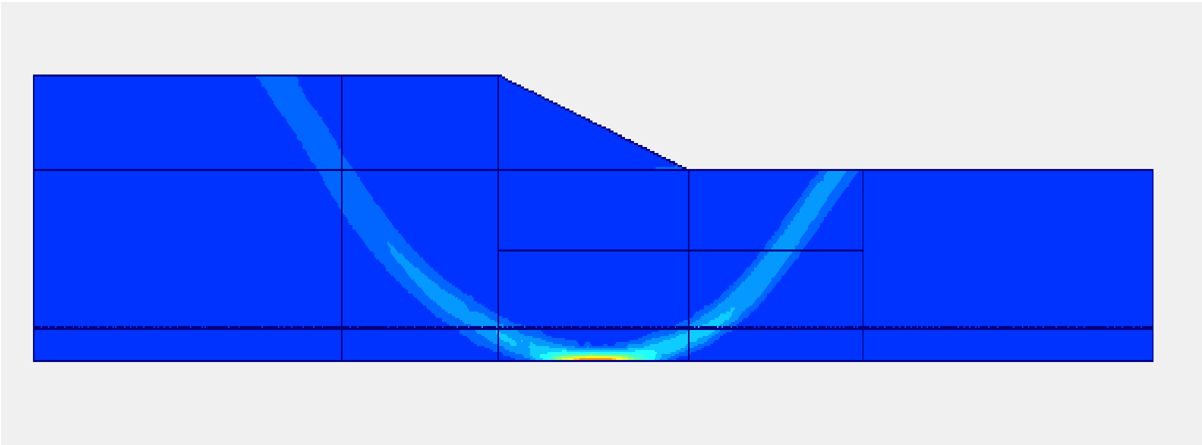


Figure 4.5: Critical Slip Surface of slope

4.1.2 Reliability-based Analysis of slope

The inclination angle of slope (β), unit weight (γ), angle of internal friction (ϕ), height of the embankment (H), undrained shear strength of foundation soil (S_u) and depth of the foundation soil (D) are taken as the input random variables for the reliability analysis. Table 4.2 gives the statistical parameters of the random variables.

Table 4.2: Statistical parameters of input random variables

Input variable	**Mean (μ)**	**Coefficient of Variation (COV %)**	**Standard deviation (σ)**
β (°)	20.0	10.0	2.0
γ (kN/m^3)	20.0	5.0	1.0
ϕ (°)	30.0	8.0	2.4

H (m)	6.0	10.0	0.6
S_u (kPa)	30.0	15.0	4.5
D (m)	12.0	10.0	1.2

The parameters are assumed as uncorrelated normally distributed. For the six input parameters Full Factorial Design is used to generate $2^k = 2^6 = 64$ sets of data. The points in the experimental design are estimated as ($\mu+\sigma$) and (μ-σ). The design has been done using Matlab (Math works 2010). FOS corresponding to 64 sets of data are analysed using PLAXIS and are tabulated (Table 4.3). The Mean μ[F], Variance V[F] and Standard Deviation σ[F] of Factors of safety are estimated using MS Excel.

Table 4.3: FOS corresponding to 64 sampling points using PLAXIS

	β	γ	ϕ	H	Su	D	FOS
μ	20	20	30	6	30	12	1.403
$\mu+\sigma$	22	21	32.4	6.6	34.5	13.2	-
$\mu-\sigma$	18	19	27.6	5.4	25.5	10.8	-
1	22	21	32.4	6.6	34.5	13.2	1.376
2	22	21	32.4	6.6	34.5	10.8	1.393
3	22	21	32.4	6.6	25.5	13.2	1.038
4	22	21	32.4	6.6	25.5	10.8	1.053
5	22	21	32.4	5.4	34.5	13.2	1.675
6	22	21	32.4	5.4	34.5	10.8	1.696
7	22	21	32.4	5.4	25.5	13.2	1.259
8	22	21	32.4	5.4	25.5	10.8	1.28
9	22	21	27.6	6.6	34.5	13.2	1.261
10	22	21	27.6	6.6	34.5	10.8	1.26
11	22	21	27.6	6.6	25.5	13.2	1.027
12	22	21	27.6	6.6	25.5	10.8	1.041
13	22	21	27.6	5.4	34.5	13.2	1.524
14	22	21	27.6	5.4	34.5	10.8	1.546
15	22	21	27.6	5.4	25.5	13.2	1.246
16	22	21	27.6	5.4	25.5	10.8	1.264
17	22	19	32.4	6.6	34.5	13.2	1.493

18	22	19	32.4	6.6	34.5	10.8	1.499
19	22	19	32.4	6.6	25.5	13.2	1.139
20	22	19	32.4	6.6	25.5	10.8	1.155
21	22	19	32.4	5.4	34.5	13.2	1.823
22	22	19	32.4	5.4	34.5	10.8	1.835
23	22	19	32.4	5.4	25.5	13.2	1.385
24	22	19	32.4	5.4	25.5	10.8	1.405
25	22	19	27.6	6.6	34.5	13.2	1.263
26	22	19	27.6	6.6	34.5	10.8	1.263
27	22	19	27.6	6.6	25.5	13.2	1.125
28	22	19	27.6	6.6	25.5	10.8	1.14
29	22	19	27.6	5.4	34.5	13.2	1.536
30	22	19	27.6	5.4	34.5	10.8	1.58
31	22	19	27.6	5.4	25.5	13.2	1.367
32	22	19	27.6	5.4	25.5	10.8	1.387
33	18	21	32.4	6.6	34.5	13.2	1.409
34	18	21	32.4	6.6	34.5	10.8	1.435
35	18	21	32.4	6.6	25.5	13.2	1.061
36	18	21	32.4	6.6	25.5	10.8	1.085
37	18	21	32.4	5.4	34.5	13.2	1.717
38	18	21	32.4	5.4	34.5	10.8	1.747
39	18	21	32.4	5.4	25.5	13.2	1.29
40	18	21	32.4	5.4	25.5	10.8	1.317
41	18	21	27.6	6.6	34.5	13.2	1.392
42	18	21	27.6	6.6	34.5	10.8	1.413
43	18	21	27.6	6.6	25.5	13.2	1.05
44	18	21	27.6	6.6	25.5	10.8	1.07
45	18	21	27.6	5.4	34.5	13.2	1.699
46	18	21	27.6	5.4	34.5	10.8	1.726
47	18	21	27.6	5.4	25.5	13.2	1.277
48	18	21	27.6	5.4	25.5	10.8	1.301
49	18	19	32.4	6.6	34.5	13.2	1.548

50	18	19	32.4	6.6	34.5	10.8	1.574
51	18	19	32.4	6.6	25.5	13.2	1.166
52	18	19	32.4	6.6	25.5	10.8	1.192
53	18	19	32.4	5.4	34.5	13.2	1.888
54	18	19	32.4	5.4	34.5	10.8	1.919
55	18	19	32.4	5.4	25.5	13.2	1.418
56	18	19	32.4	5.4	25.5	10.8	1.446
57	18	19	27.6	6.6	34.5	13.2	1.511
58	18	19	27.6	6.6	34.5	10.8	1.525
59	18	19	27.6	6.6	25.5	13.2	1.152
60	18	19	27.6	6.6	25.5	10.8	1.174
61	18	19	27.6	5.4	34.5	13.2	1.843
62	18	19	27.6	5.4	34.5	10.8	1.865
63	18	19	27.6	5.4	25.5	13.2	1.404
64	18	19	27.6	5.4	25.5	10.8	1.428
						μ[F]=	1.411325
						σ[F] =	0.202075
						Var[F] =	0.040196

To get a linear response surface model, regression analysis is carried out using the 64 factors obtained from FEM. Least Square Error method is adopted for Regression analysis (MS Excel).

$$FOS = 2.841083 + (-0.021156 * \beta) + (-0.055 * \gamma) + (0.013385 * \phi) + (-0.229427 * H) + (0.038514 * S_u) + (-0.00849 * D)$$

(R^2= 0.949, R^2_{adj}= 0.944)

For conservative deformation behaviour, the correlation coefficients are taken as zero. A linear correlation model for the parameters is assumed and the performance function is defined as

g(x) = FOS -1

By using MS Excel's built-in Solver optimization tool, the reliability index,

$$\beta = min\sqrt{\left[\frac{x_i\text{-}\mu_i}{\sigma_i}\right]^T\left[\frac{x_i\text{-}\mu_i}{\sigma_i}\right]}$$

is minimized with constraint that g(x) = 0 by changing the values of random variables, x_i. Initially the value of x_i is assumed nearer to the mean value of input parameter.

$\beta = 1.692$

The Probability of Failure (P_f) is estimated from the reliability index β as follows:

$P_f = 1- \Phi(\beta) = \Phi(-\beta) = \Phi(-1.692)$

From the excel, P_f = NORMSDIST(-1.692) gives $P_f = 0.045357 = 4.54$ %

4.2 Limit Analysis Method using LimitState:GEO

4.2.1 Deterministic analysis of slope

The slope is modelled using LimitState:GEO 3.2.d as shown in Figure 4.6 Mohr-coulomb model is considered to represent the soil. Nodal density is taken as very fine (2000 nodes). The soil parameters and the dimensions are taken from Table 4.1 and Figure 4.1 respectively.

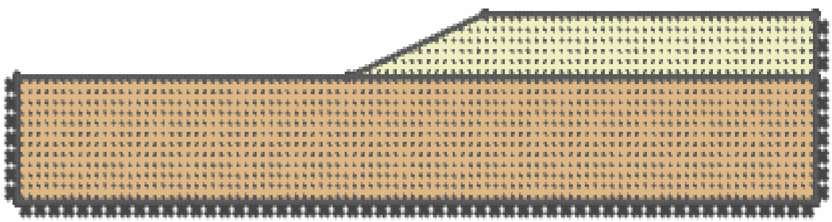


Figure 4.6: LimitState:GEO model of slope

Factor of safety is indicated in terms of Adequacy factor for the Factor Strength. The failure mechanism and deformation of the slope is shown in Figure 4.7 and Figure 4.8 respectively.

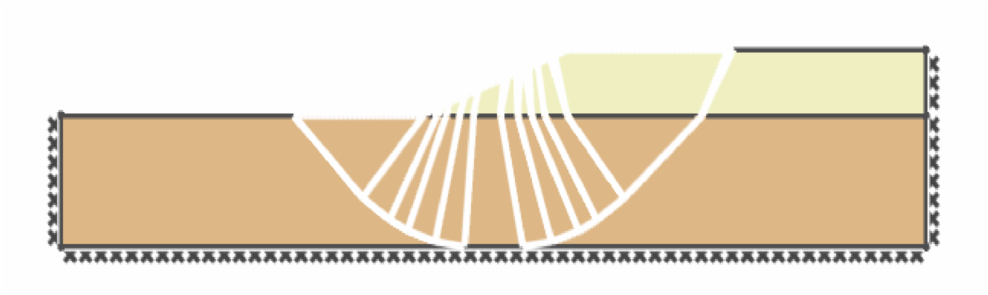


Figure 4.7: Failure Mechanism in Slope

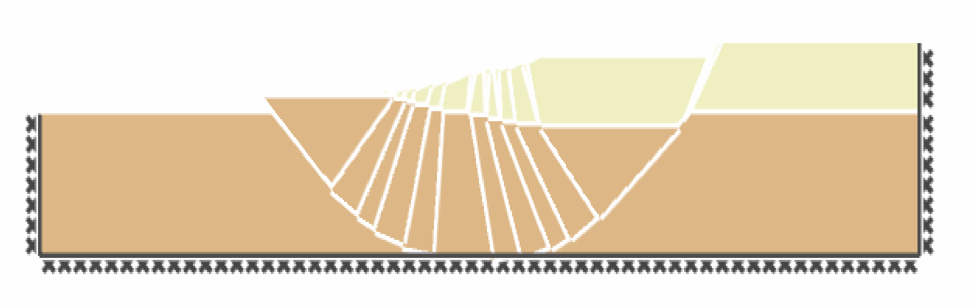


Figure 4.8: Deformed Slope

4.2.2 Reliability-based analysis of slope

The FOS corresponding to the 64 sample points using LimitState:GEO along with its mean value μ[F], Standard deviation σ[F] and variance *Var*[F]are shown in Table 4.4.

Table 4.4: FOS corresponding to 64 sampling points using LimitState:GEO

	β	γ	ϕ	H	S_u	D	FOS
μ	20	20	30	6	30	12	1.416
COV %	10	5	8	10	15	10	-
σ	2	1	2.4	0.6	4.5	1.2	-
$\mu+\sigma$	22	21	32.4	6.6	34.5	13.2	-
$\mu-\sigma$	18	19	27.6	5.4	25.5	10.8	-
1	22	21	32.4	6.6	34.5	13.2	1.386
2	22	21	32.4	6.6	34.5	10.8	1.404
3	22	21	32.4	6.6	25.5	13.2	1.056
4	22	21	32.4	6.6	25.5	10.8	1.076
5	22	21	32.4	5.4	34.5	13.2	1.769
6	22	21	32.4	5.4	34.5	10.8	1.709

7	22	21	32.4	5.4	25.5	13.2	1.278
8	22	21	32.4	5.4	25.5	10.8	1.3
9	22	21	27.6	6.6	34.5	13.2	1.406
10	22	21	27.6	6.6	34.5	10.8	1.43
11	22	21	27.6	6.6	25.5	13.2	1.039
12	22	21	27.6	6.6	25.5	10.8	1.057
13	22	21	27.6	5.4	34.5	13.2	1.498
14	22	21	27.6	5.4	34.5	10.8	1.3
15	22	21	27.6	5.4	25.5	13.2	1.257
16	22	21	27.6	5.4	25.5	10.8	1.276
17	22	19	32.4	6.6	34.5	13.2	1.484
18	22	19	32.4	6.6	34.5	10.8	1.623
19	22	19	32.4	6.6	25.5	13.2	1.588
20	22	19	32.4	6.6	25.5	10.8	1.2
21	22	19	32.4	5.4	34.5	13.2	1.814
22	22	19	32.4	5.4	34.5	10.8	2.003
23	22	19	32.4	5.4	25.5	13.2	1.4
24	22	19	32.4	5.4	25.5	10.8	1.423
25	22	19	27.6	6.6	34.5	13.2	1.554
26	22	19	27.6	6.6	34.5	10.8	1.554
27	22	19	27.6	6.6	25.5	13.2	1.133
28	22	19	27.6	6.6	25.5	10.8	1.333
29	22	19	27.6	5.4	34.5	13.2	1.5
30	22	19	27.6	5.4	34.5	10.8	1.959
31	22	19	27.6	5.4	25.5	13.2	1.378
32	22	19	27.6	5.4	25.5	10.8	1.397
33	18	21	32.4	6.6	34.5	13.2	1.428
34	18	21	32.4	6.6	34.5	10.8	1.529
35	18	21	32.4	6.6	25.5	13.2	1.088

36	18	21	32.4	6.6	25.5	10.8	1.114
37	18	21	32.4	5.4	34.5	13.2	1.738
38	18	21	32.4	5.4	34.5	10.8	1.772
39	18	21	32.4	5.4	25.5	13.2	1.423
40	18	21	32.4	5.4	25.5	10.8	1.352
41	18	21	27.6	6.6	34.5	13.2	1.411
42	18	21	27.6	6.6	34.5	10.8	1.439
43	18	21	27.6	6.6	25.5	13.2	1.112
44	18	21	27.6	6.6	25.5	10.8	1.123
45	18	21	27.6	5.4	34.5	13.2	1.736
46	18	21	27.6	5.4	34.5	10.8	1.812
47	18	21	27.6	5.4	25.5	13.2	1.322
48	18	21	27.6	5.4	25.5	10.8	1.368
49	18	19	32.4	6.6	34.5	13.2	1.612
50	18	19	32.4	6.6	34.5	10.8	1.655
51	18	19	32.4	6.6	25.5	13.2	1.234
52	18	19	32.4	6.6	25.5	10.8	1.358
53	18	19	32.4	5.4	34.5	13.2	1.925
54	18	19	32.4	5.4	34.5	10.8	2.03
55	18	19	32.4	5.4	25.5	13.2	1.588
56	18	19	32.4	5.4	25.5	10.8	1.497
57	18	19	27.6	6.6	34.5	13.2	1.62
58	18	19	27.6	6.6	34.5	10.8	1.651
59	18	19	27.6	6.6	25.5	13.2	1.253
60	18	19	27.6	6.6	25.5	10.8	1.299
61	18	19	27.6	5.4	34.5	13.2	1.997
62	18	19	27.6	5.4	34.5	10.8	2.05
63	18	19	27.6	5.4	25.5	13.2	1.448
64	18	19	27.6	5.4	25.5	10.8	1.496

						μ[Fs]=	1.469
						σ[Fs] =	0.261
						Var[Fs] =	0.068

Using the FOS values obtained, regression analysis is done for the response surface model,

$$FOS = 3.645375 + (-0.022625 * \beta) + (-0.0945 * \gamma) + (0.010729 * \phi) + (-0.197031 * H) + (0.040042 * S_u) + (-0.014505 * D)$$

(R^2= 0.873, R^2_{adj}= 0.86)

Solving for the reliability index using solver optimization tool, the value obtained is:

β = 1.944

The probability of failure $P_f = \Phi(-1.944)$ = 0.0259 = 2.59 %

4.3 Analytical method

The analytical method given by Low, 1989 for the embankment on soft ground is used for this study. The embankment is shown in Figure 4.9. Slope angle β, height H, angle of internal friction ϕ_m and cohesion C_m, and unit weight γ characterizes the embankment. Undrained shear strength C_A characterizes the foundation soil. It is assumed that the angle of internal friction is zero for the foundation materials. The horizontal line below the top of foundation at a depth, D is the Trial Limiting Tangent to which the potential slip surfaces are tangential.

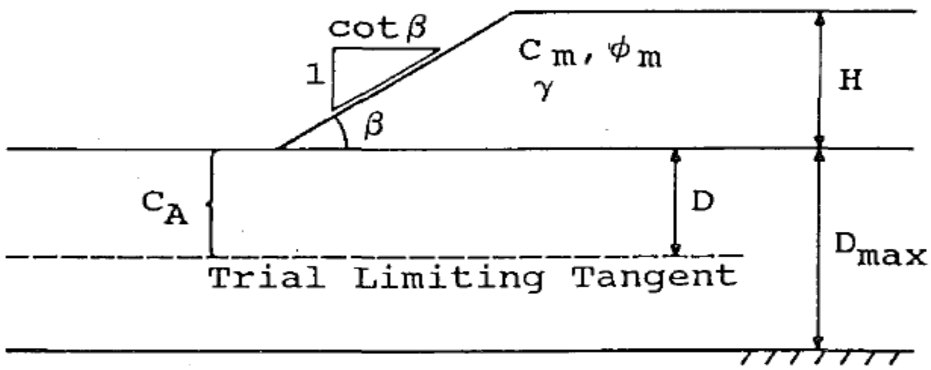


Figure 4.9: Geometry of embankment on soft soil

The factor of safety equation is given as:

$$(F_s)_D = N_1\left(\frac{C_A}{\gamma H}\right) + N_2\left(\frac{C_m}{\gamma H} + \lambda \tan \emptyset_m\right) \ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Where $N_1 = 3.06\left(\frac{D}{H}\right)^{0.53}\left(\frac{{\alpha_1}^{1.47}}{\alpha_2}\right)$

$$N_2 = 1.53\left[\left(\frac{D}{H}+1\right)^{0.53} - \left(\frac{D}{H}\right)^{0.53}\right]\left(\frac{{\alpha_1}^{1.47}}{\alpha_2}\right)$$

$$\alpha_1 = 1.564\left(\frac{D}{H}+\frac{1}{2}\right) + 0.1303\left(\frac{cot^2\beta + 1}{\frac{D}{H}+0.5}\right)$$

$$\alpha_2 = \alpha_1\left(\frac{D}{H}+\frac{1}{2}\right) - \frac{1}{2}\left(\frac{D}{H}+\frac{1}{2}\right)^2 - \frac{1}{24}(cot^2\beta + 1)$$

$$\lambda \approx 0.19 + \frac{0.02 \cot\beta}{D/H} \qquad \text{(For D/H} \geq 0.5)$$

N1 and N2 are the stability numbers for normalized foundation strength and normalized embankment strength, respectively.

4.3.1 Deterministic Analysis of slope

For the slope in present study, the values assigned to each parameter in the equation are as follows:

$\beta = 20^o$; $C_m = 0$ kPa ; $\phi_m = 30^o$; H = 12m ; $C_A = 30$ kPa ; D = 12m

Substituting the above values in the FOS equation, the obtained FOS is 1.408 with critical slip surface located at (x, y) = (8.25, 4.36). The critical slip circle of the slope is shown in Figure 4.10

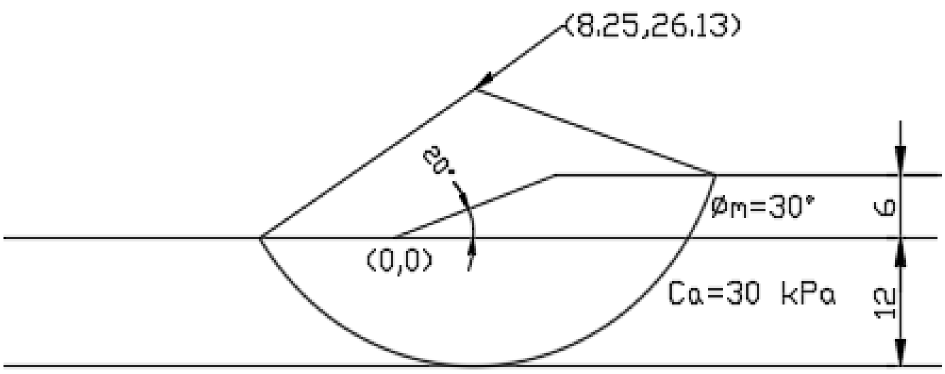


Figure 4.10: Critical slip circle using analytical method

4.3.2 Reliability-based Analysis of slope

The FOS along with stability numbers N1 and N2 corresponding to 64 sampling points are presented in Table 4.5 using the analytical procedure.

Table 4.5: FOS corresponding to 64 sampling points using Analytical method

	β	γ	ϕ	H	Su	D	N_1	N_2	FOS
μ	20	20	30	6	30	12	5.188	0.622	1.408
COV %	10	5	8	10	15	10	-	-	-
σ	2	1	2.4	0.6	4.5	1.2	-	-	-
$\mu+\sigma$	22	21	32.4	6.6	34.5	13.2	-	-	-
$\mu-\sigma$	18	19	27.6	5.4	25.5	10.8	-	-	-
1	22	21	32.4	6.6	34.5	13.2	5.146	0.617	1.594
2	22	21	32.4	6.6	34.5	10.8	5.104	0.734	1.598
3	22	21	32.4	6.6	25.5	13.2	5.146	0.617	1.198
4	22	21	32.4	6.6	25.5	10.8	5.104	0.734	1.206
5	22	21	32.4	5.4	34.5	13.2	5.188	0.517	1.593
6	22	21	32.4	5.4	34.5	10.8	5.146	0.617	1.594
7	22	21	32.4	5.4	25.5	13.2	5.188	0.517	1.193
8	22	21	32.4	5.4	25.5	10.8	5.146	0.617	1.198
9	22	21	27.6	6.6	34.5	13.2	5.146	0.617	1.594
10	22	21	27.6	6.6	34.5	10.8	5.104	0.734	1.598
11	22	21	27.6	6.6	25.5	13.2	5.146	0.617	1.198
12	22	21	27.6	6.6	25.5	10.8	5.104	0.734	1.206
13	22	21	27.6	5.4	34.5	13.2	5.188	0.517	1.593
14	22	21	27.6	5.4	34.5	10.8	5.146	0.617	1.594
15	22	21	27.6	5.4	25.5	13.2	5.188	0.517	1.193
16	22	21	27.6	5.4	25.5	10.8	5.146	0.617	1.198
17	22	19	32.4	6.6	34.5	13.2	5.146	0.617	1.594
18	22	19	32.4	6.6	34.5	10.8	5.104	0.734	1.598
19	22	19	32.4	6.6	25.5	13.2	5.146	0.617	1.198
20	22	19	32.4	6.6	25.5	10.8	5.104	0.734	1.206

21	22	19	32.4	5.4	34.5	13.2	5.188	0.517	1.593
22	22	19	32.4	5.4	34.5	10.8	5.146	0.617	1.594
23	22	19	32.4	5.4	25.5	13.2	5.188	0.517	1.193
24	22	19	32.4	5.4	25.5	10.8	5.146	0.617	1.198
25	22	19	27.6	6.6	34.5	13.2	5.146	0.617	1.594
26	22	19	27.6	6.6	34.5	10.8	5.104	0.734	1.598
27	22	19	27.6	6.6	25.5	13.2	5.146	0.617	1.198
28	22	19	27.6	6.6	25.5	10.8	5.104	0.734	1.206
29	22	19	27.6	5.4	34.5	13.2	5.188	0.517	1.593
30	22	19	27.6	5.4	34.5	10.8	5.146	0.617	1.594
31	22	19	27.6	5.4	25.5	13.2	5.188	0.517	1.193
32	22	19	27.6	5.4	25.5	10.8	5.146	0.617	1.198
33	18	21	32.4	6.6	34.5	13.2	5.244	0.629	1.626
34	18	21	32.4	6.6	34.5	10.8	5.232	0.752	1.642
35	18	21	32.4	6.6	25.5	13.2	5.244	0.629	1.223
36	18	21	32.4	6.6	25.5	10.8	5.232	0.752	1.239
37	18	21	32.4	5.4	34.5	13.2	5.261	0.524	1.616
38	18	21	32.4	5.4	34.5	10.8	5.244	0.629	1.626
39	18	21	32.4	5.4	25.5	13.2	5.261	0.524	1.212
40	18	21	32.4	5.4	25.5	10.8	5.244	0.629	1.223
41	18	21	27.6	6.6	34.5	13.2	5.244	0.629	1.626
42	18	21	27.6	6.6	34.5	10.8	5.232	0.752	1.642
43	18	21	27.6	6.6	25.5	13.2	5.244	0.629	1.223
44	18	21	27.6	6.6	25.5	10.8	5.232	0.752	1.239
45	18	21	27.6	5.4	34.5	13.2	5.261	0.524	1.616
46	18	21	27.6	5.4	34.5	10.8	5.244	0.629	1.626
47	18	21	27.6	5.4	25.5	13.2	5.261	0.524	1.212
48	18	21	27.6	5.4	25.5	10.8	5.244	0.629	1.223
49	18	19	32.4	6.6	34.5	13.2	5.244	0.629	1.626
50	18	19	32.4	6.6	34.5	10.8	5.232	0.752	1.642
51	18	19	32.4	6.6	25.5	13.2	5.244	0.629	1.223
52	18	19	32.4	6.6	25.5	10.8	5.232	0.752	1.239

53	18	19	32.4	5.4	34.5	13.2	5.261	0.524	1.616
54	18	19	32.4	5.4	34.5	10.8	5.244	0.629	1.626
55	18	19	32.4	5.4	25.5	13.2	5.261	0.524	1.212
56	18	19	32.4	5.4	25.5	10.8	5.244	0.629	1.223
57	18	19	27.6	6.6	34.5	13.2	5.244	0.629	1.626
58	18	19	27.6	6.6	34.5	10.8	5.232	0.752	1.642
59	18	19	27.6	6.6	25.5	13.2	5.244	0.629	1.223
60	18	19	27.6	6.6	25.5	10.8	5.232	0.752	1.239
61	18	19	27.6	5.4	34.5	13.2	5.261	0.524	1.616
62	18	19	27.6	5.4	34.5	10.8	5.244	0.629	1.626
63	18	19	27.6	5.4	25.5	13.2	5.261	0.524	1.212
64	18	19	27.6	5.4	25.5	10.8	5.244	0.629	1.223
								μ[F] =	1.411
								σ[F] =	0.202
								Var[F] =	0.041

Using the FOS values obtained, regression analysis is done for the response surface model,

$$FOS = 0.22496 + (-0.00729 * \beta) + (3.06102E\text{-}18 * \gamma) + (1.25808E\text{-}18 * \phi) + (0.00736 * H) + (0.04441 * S_u) + (-0.00368 * D)$$

($R^2 = 0.999$, $R^2_{adj} = 0.999$)

Solving for the reliability index using solver optimization tool, the value obtained is:

$\beta = 2.052$

The probability of failure $P_f = \Phi(-2.052) = 0.02009 = 2.01$ %

The FOS, reliability index (β) and probability of failure (P_f) values of the slope obtained from above three analyses and from literature are tabulated (Table 4.6) as follows:

Table 4.6: Comparison of outputs obtained from different analyses for the slope

Method of Analysis	**Fs**		$\boldsymbol{\beta}$	$\mathbf{P_f}$ **%**
	Deterministic approach	**Reliability-based approach**		
Finite Element Method	1.40	1.41	1.692	4.54
Limit Analysis Method	1.42	1.47	1.944	2.56

Analytical method	1.41	1.41	2.052	2.01

4.3.3 Parametric study

This study will investigate the stability of c-ϕ embankment fill placed on soft clay using the analytical method given by Low, 1989 and Finite Element Analysis using the software PLAXIS. The D/H values are varied as 0.5, 1, 2, 3, 4, 5 and cot β values are varied as 1, 2, 3, 4, 5. For each value of cot β, the D/H values are varied and the corresponding FOS values are calculated using the equation given by Low, 1989 keeping the undrained shear strength of foundation soil C_A = 30 kPa and that of embankment fill, C_m = 15 kPa as constant. This process is repeated for other values of cot β. The angle of internal friction of the embankment soil, ϕ_m is taken as 30º. The plot of FOS - D/H is made for different values of cot β as shown in Figure 4.11.

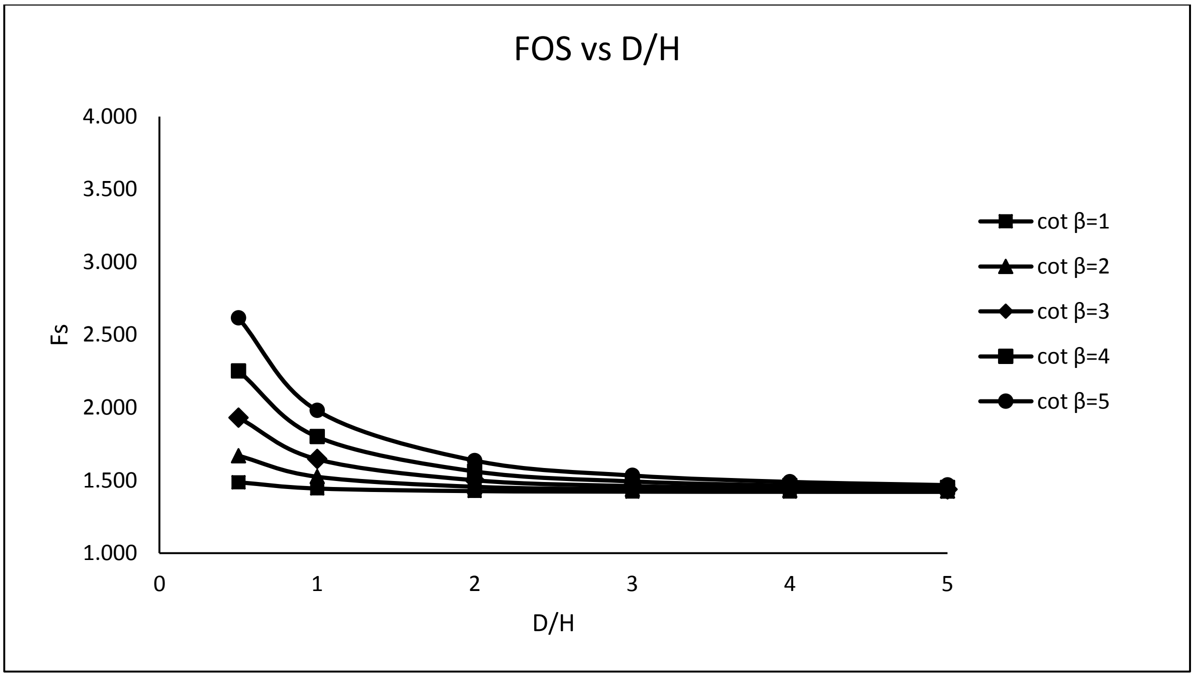


Figure 4.11: FOS Vs D/H plot for different values of cot β using Low Equation

Using PLAXIS, the FOS values are determined for slope varying D/H values for different cot β. The FOS for varying D/H and cot β are listed in Table 4.6 (a, b, c, d, e). The plot of FOS-

D/H is made using these FOS values and compared with those obtained from analytical study. Figure 4.12 shows the comparison of the plots.

Table 4.6: FOS corresponding to different D/H values using Low's Eqn. and FEM

(a) cot β = 1

D/H	N_1	N_2	λ	FOS using Low's Eqn	FOS using FEM
0.5	4.133	1.633	0.230	1.486	1.799
1	4.630	1.028	0.210	1.443	1.724
2	4.992	0.598	0.200	1.426	1.668
3	5.147	0.424	0.197	1.422	1.646
4	5.234	0.329	0.195	1.421	1.633
5	5.290	0.268	0.194	1.421	1.625

(b) cot β = 2

D/H	N_1	N_2	λ	FOS using Low's Eqn	FOS using FEM
0.5	4.527	1.789	0.270	1.669	2.095
1	4.849	1.076	0.230	1.524	1.890
2	5.083	0.609	0.210	1.455	1.761
3	5.196	0.428	0.203	1.437	1.710
4	5.264	0.330	0.200	1.430	1.685
5	5.311	0.269	0.198	1.427	1.667

(c) cot β = 3

D/H	N_1	N_2	λ	FOS using Low's Eqn	FOS using FEM
0.5	5.111	2.019	0.310	1.931	2.473
1	5.192	1.152	0.250	1.645	2.088
2	5.230	0.627	0.220	1.501	1.843
3	5.275	0.434	0.210	1.461	1.768
4	5.314	0.334	0.205	1.445	1.727
5	5.345	0.271	0.202	1.437	1.699

(d) cot β = 4

D/H	N_1	N_2	λ	FOS using Low's Eqn	FOS using FEM
0.5	5.819	2.299	0.350	2.251	2.859
1	5.632	1.250	0.270	1.799	2.316
2	5.428	0.651	0.230	1.562	1.947
3	5.385	0.443	0.217	1.493	1.828
4	5.383	0.338	0.210	1.464	1.772
5	5.392	0.273	0.206	1.450	1.736

(e) cot β = 5

D/H	N_1	N_2	λ	FOS using Low's Eqn	FOS using FEM
0.5	6.608	2.610	0.390	2.617	3.234
1	6.147	1.364	0.290	1.979	2.576
2	5.671	0.680	0.240	1.636	2.069
3	5.522	0.455	0.223	1.533	1.894
4	5.469	0.343	0.215	1.489	1.819
5	5.451	0.277	0.210	1.467	1.775

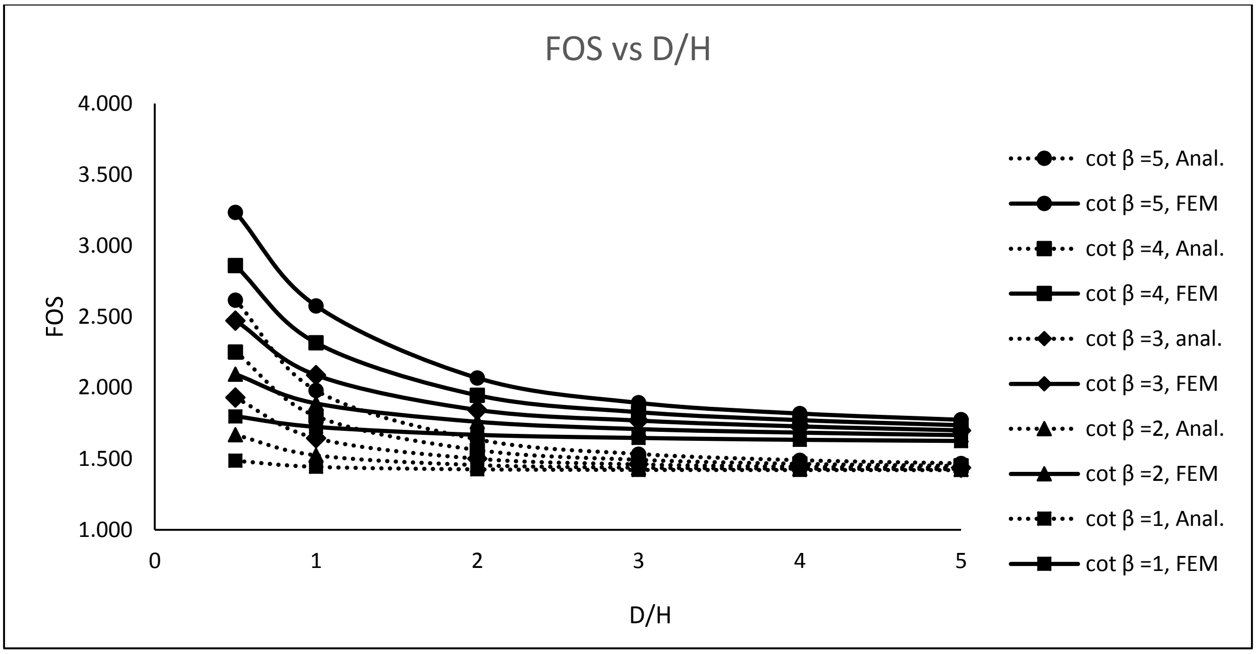


Figure 4.12: FOS Vs D/H plot for different values of cot β using Low Equation and FEM

From the Figure 4.12 it can be observed that the FOS values obtained from the Finite Element Analysis are higher than that from the Analytical Method. The FOS values are investigated and a factor k = 1.2 is multiplied to N_1, N_2, λ values in Low's equation to obtain FOS values closer to FEM. Thus the Modified Low's equation of Factor of Safety is obtained as:

$$(F_s)_D{}' = N_1{}'\left(\frac{C_A}{\gamma H}\right) + N_2{}'\left(\frac{C_m}{\gamma H} + \lambda' \tan\emptyset_m\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Where $N_1{}' = \mathbf{1.2} * \left\{3.06\left(\frac{D}{H}\right)^{0.53}\left(\frac{{\alpha_1}^{1.47}}{\alpha_2}\right)\right\}$

$$N_2{}' = \mathbf{1.2} * \left\{1.53\left[\left(\frac{D}{H}+1\right)^{0.53} - \left(\frac{D}{H}\right)^{0.53}\right]\left(\frac{{\alpha_1}^{1.47}}{\alpha_2}\right)\right\}$$

$$\alpha_1 = 1.564\left(\frac{D}{H}+\frac{1}{2}\right) + 0.1303\left(\frac{cot^2\beta+1}{\frac{D}{H}+0.5}\right)$$

$$\alpha_2 = \alpha_1\left(\frac{D}{H}+\frac{1}{2}\right) - \frac{1}{2}\left(\frac{D}{H}+\frac{1}{2}\right)^2 - \frac{1}{24}(cot^2\beta+1)$$

$$\lambda' \approx \mathbf{1.2} * \left\{0.19 + \frac{0.02\cot\beta}{D/H}\right\} \qquad \text{(For D/H} \geq 0.5\text{)}$$

N_1' and N_2' are the modified stability numbers for normalized foundation strength and normalized embankment strength, respectively and λ' is the modified factor.

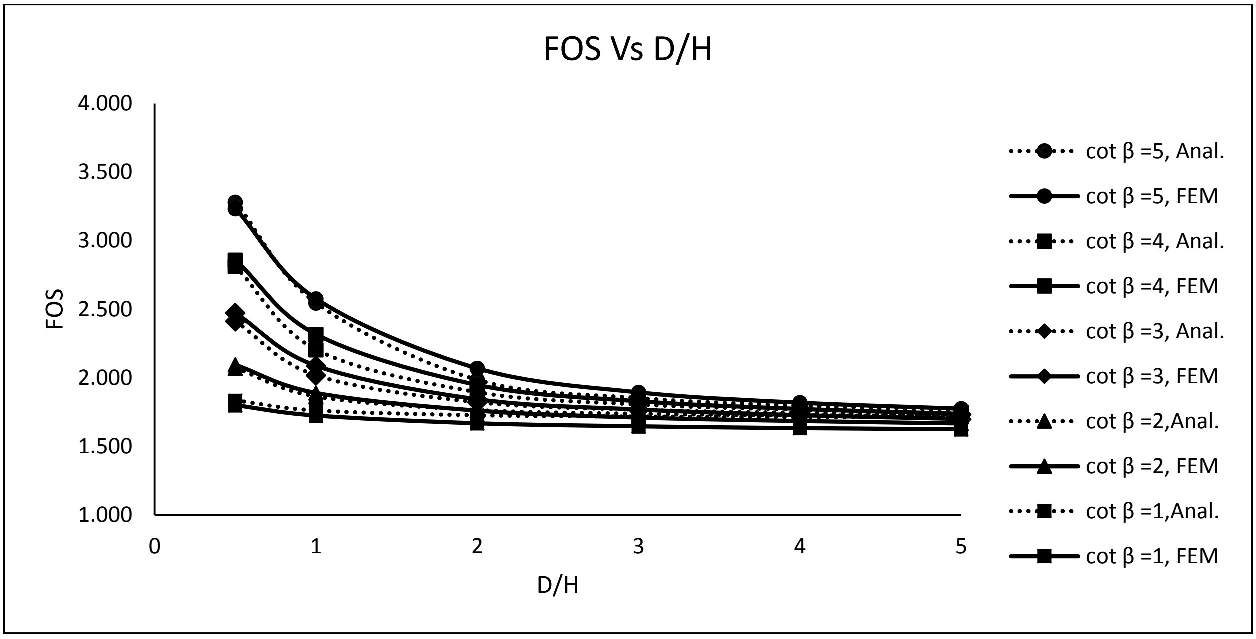


Figure 4.13: FOS vs D/H plot for different values of cot β using Modified Low's Equation and FEM

The Modified stability numbers N_1', N_2', Modified factor λ', and corresponding Modified Factors of Safety for varying D/H values for different cot β and for ϕ = 22.5º, 30º, 45º are calculated and listed in Table 4.7

Table 4.7: Modified FOS corresponding to different D/H values using Modified Low's Eqn.

(a) cot β = 1

D/H	**N_1'**	**N_2'**	**λ'**	**FOS'**		
				ϕ = 22.5º	**ϕ = 30º**	**ϕ = 45º**
0.5	4.959	1.959	0.276	1.747	1.835	2.063
1	5.556	1.233	0.252	1.711	1.762	1.893
2	5.991	0.718	0.240	1.700	1.728	1.801
3	6.177	0.509	0.236	1.699	1.718	1.769
4	6.281	0.394	0.234	1.699	1.714	1.753
5	6.349	0.322	0.233	1.700	1.712	1.744

(b) cot β = 2

D/H	**N_1'**	**N_2'**	**λ'**	**FOS'**		
				ϕ = 22.5º	**ϕ = 30º**	**ϕ = 45º**
0.5	5.433	2.146	0.324	1.956	2.070	2.364
1	5.819	1.292	0.276	1.805	1.863	2.014
2	6.100	0.731	0.252	1.734	1.764	1.842
3	6.235	0.513	0.244	1.716	1.737	1.790
4	6.317	0.397	0.240	1.710	1.726	1.766
5	6.373	0.323	0.238	1.707	1.720	1.752

(c) cot β = 3

D/H	**N_1'**	**N_2'**	**λ'**	**FOS'**		
				ϕ = 22.5º	**ϕ = 30º**	**ϕ = 45º**
0.5	6.133	2.423	0.378	2.263	2.412	2.799
1	6.230	1.383	0.305	1.949	2.018	2.196
2	6.276	0.752	0.268	1.789	1.822	1.908
3	6.331	0.521	0.256	1.745	1.767	1.824
4	6.377	0.400	0.250	1.728	1.744	1.787
5	6.414	0.325	0.246	1.719	1.732	1.766

(d) cot β = 4

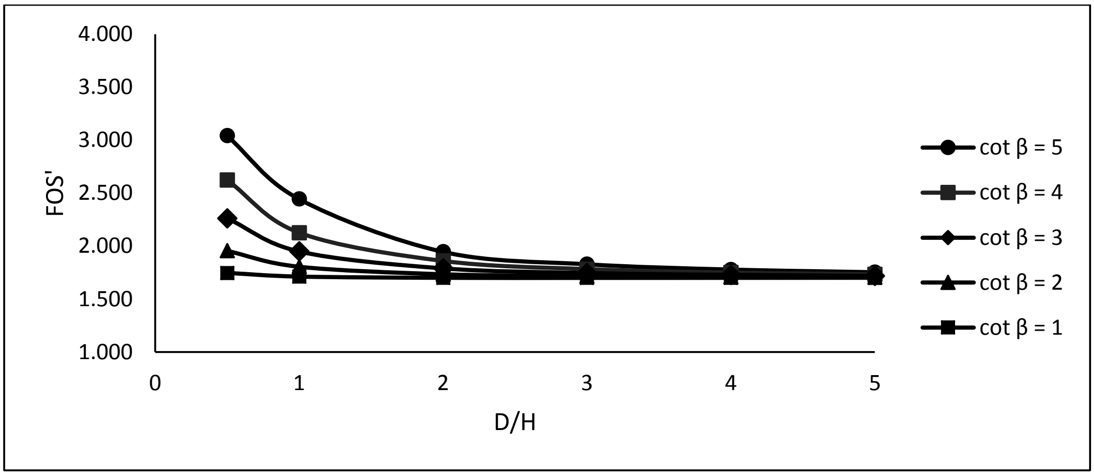
D/H	**N_1'**	**N_2'**	**λ'**	**FOS'**		
				ϕ = 22.5º	**ϕ = 30º**	**ϕ = 45º**
0.5	6.983	2.759	0.420	2.624	2.813	3.303
1	6.758	1.500	0.324	2.127	2.206	2.411
2	6.514	0.781	0.276	1.860	1.895	1.986
3	6.462	0.532	0.260	1.782	1.805	1.863
4	6.459	0.405	0.252	1.751	1.767	1.810

5	6.470	0.328	0.247	1.735	1.748	1.782

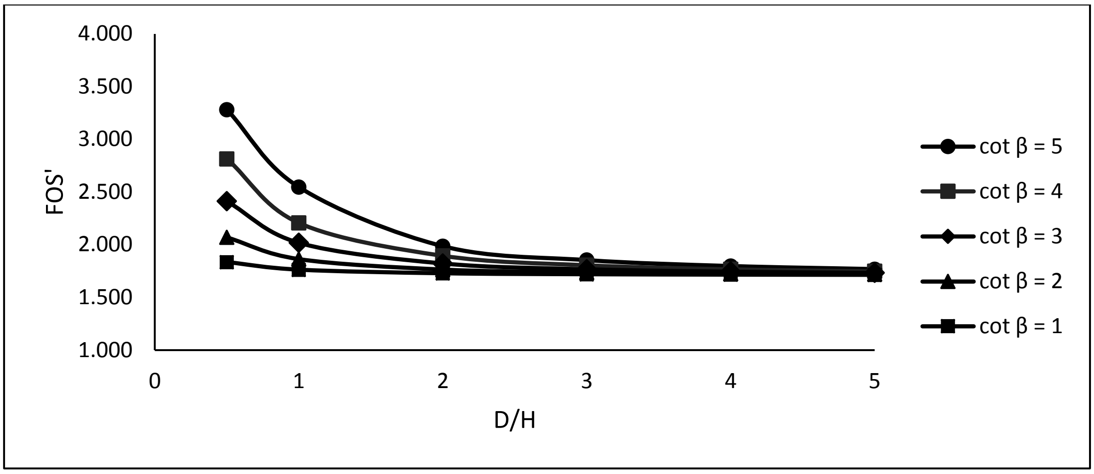
(e) cot β = 5

D/H	N1'	N2'	λ'	FOS'		
				ϕ = 30º	ϕ = 45º	ϕ = 22.5º
0.5	7.929	3.132	0.468	3.042	3.281	3.901
1	7.683	1.705	0.363	2.445	2.546	2.807
2	6.806	0.816	0.288	1.947	1.985	2.085
3	6.626	0.546	0.268	1.830	1.853	1.915
4	6.563	0.412	0.258	1.780	1.797	1.842
5	6.541	0.332	0.252	1.754	1.768	1.803

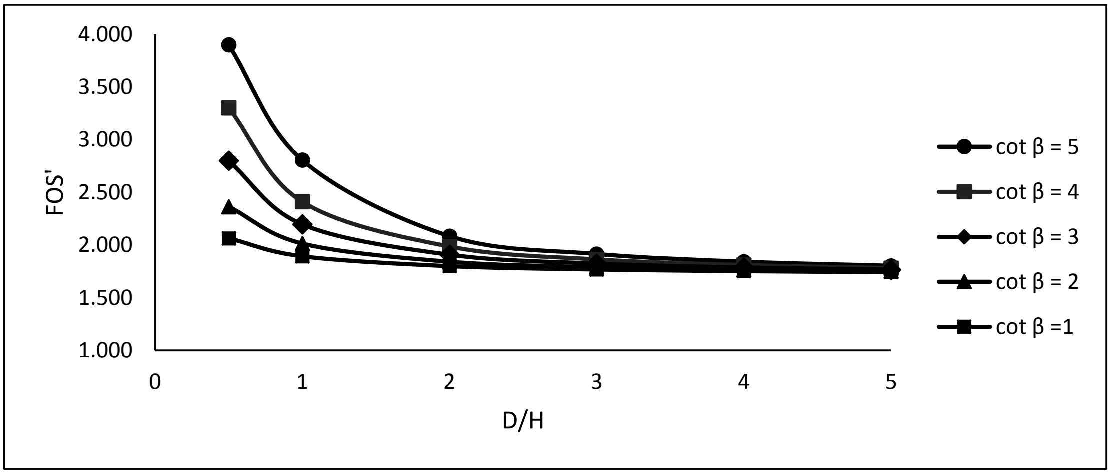
The Charts of FOS vs D/H for different values of cot β corresponding to ϕ = 22.5º, 30º, 45º are prepared for the modified Low's Equation of Factor of Safety and are presented in Figure 4.14.



(a) ϕ = 22.5º



(b) $\phi = 30^{o}$



(c) $\phi = 45^{o}$

Figure 4.14: Charts of FOS vs D/H for different values of cot β

Chapter 5

Case study

Analysis of dykes of the James Bay hydroelectric project

This example is drawn from the slope stability analysis of James Bay Dykes described by El-Ramly et al. (2002). It is a hydroelectric project in Northern Quebec, Canada. The uncertainties and spatial variability in the soil properties have been documented by Ladd (1983 and 1991). Christian et al. (1994) used this data for doing a probabilistic stability analysis. The stratigraphy and cross section of James Bay dykes are shown in Figure 5.1. It is an embankment constructed in single stage of 12m height with the slope angle of 18.43° (3:1). A berm of 56m is at the mid-height. The embankment is on about 4m thick clay crust which in turn overlies on marine clay of about 8m thickness. The sensitive marine clay is underlain by lacustrine clay of about 6.5m thickness. The marine clay has an undrained shear strength of about 34.5 kPa and that of lacustrine clay is about 31.2 kPa. The lacustrine clay is on stiff till.

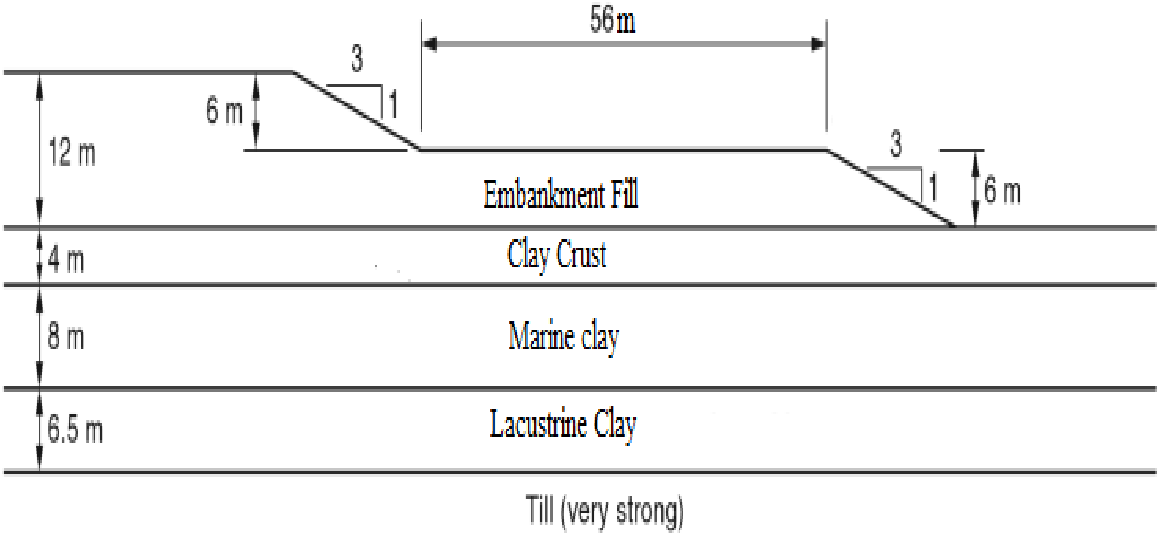


Figure 5.1: Stratigraphy and cross section of James Bay dykes

5.1 Finite Element Method

5.1.1 Deterministic Analysis

The slope is modelled using the available software PLAXIS 2D-V9.02 as shown in Figure 5.2. Full fixity is considered at the bottom of the foundation soil and horizontal fixity at the sides of the model. The soil is represented using simple elastic, perfectly plastic Mohr-Coulomb model. The soil is modelled using 15-noded triangular elements with the 12-point integration rule. Soil parameters used for this case are listed in Table 5.1. In this case, the Young's modulus E is taken as 100 MPa for the embankment fill. For the soft clay in the foundation, the ratio of the undrained modulus, E_u, to undrained shear strength, S_u, is chosen to be 1000 (Duncan and Buchignani, 1976). Poisson's ratio, υ is considered as 0.3 for all the foundation soils.

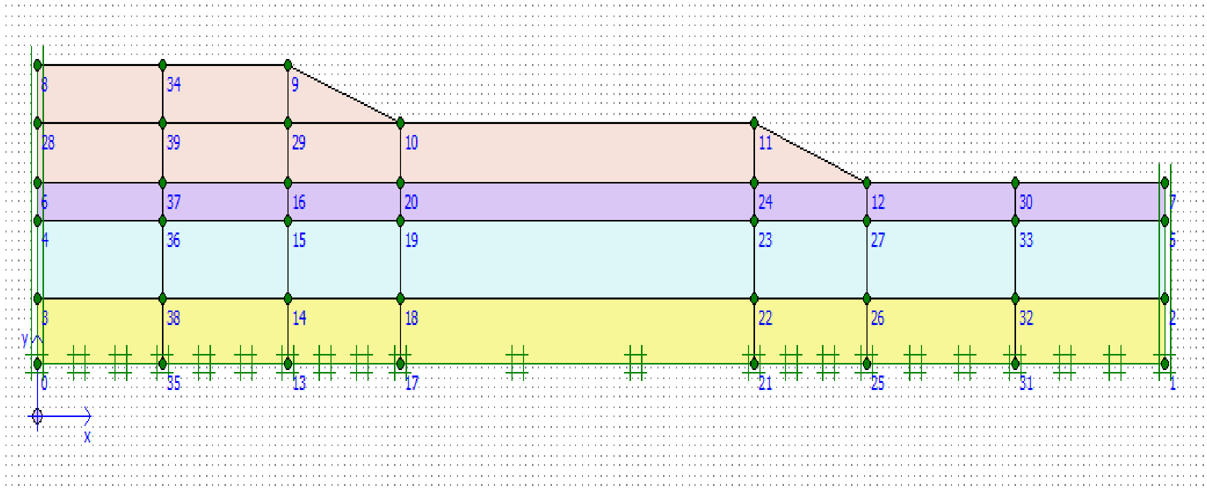


Figure 5.2: PLAXIS model

Table 5.1: Soil Parameters of embankment and foundation soil of James Bay dyke

	Unit weight γ **(kN/m^3)**	**Friction Angle** ϕ **(°)**	**Shear strength** S_u **(kN/m^2)**
Embankment fill	20	30	0

Clay crust	19	0	41
Marine clay	19	0	34.5
Lacustrine clay	20.5	0	31.2

The mesh is generated using Fine coarseness globally. Clusters are formed in the critical areas of the slope and foundation. These clusters are refined further to increase the no. of elements using cluster refinement. The lines forming the boundaries of the clusters are also refined using the Line refinement. The refinement around the crest and toe nodes of the slope is done using

Point refinement. The meshing details are shown in Figure 5.3.

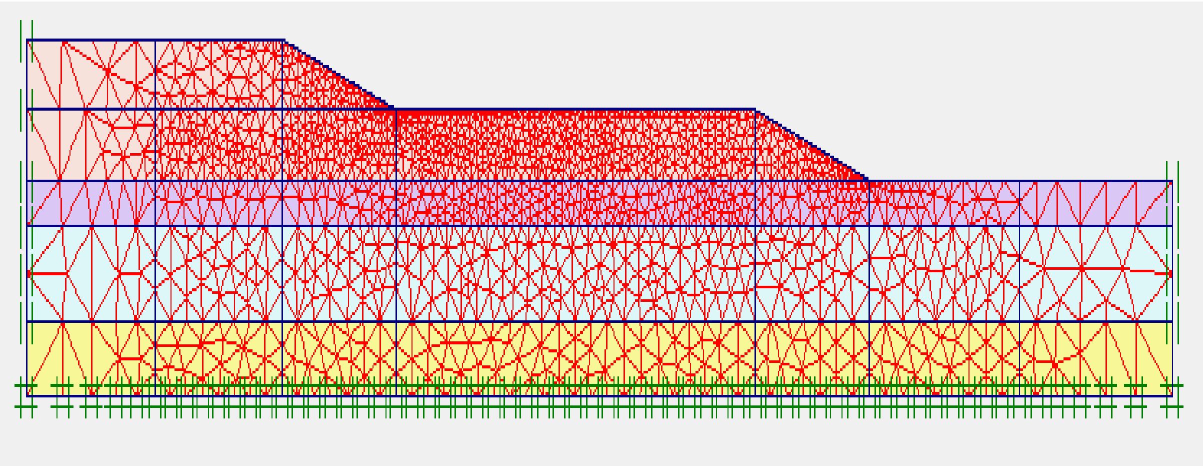


Figure 5.3: Meshing details of James Bay case

Phi/c reduction method in PLAXIS is used for calculating the Factor of Safety. It is represented as sum of incremental multiplier, ΣMsf and is defined as the ratio of the available shear strength to the shear strength at failure. Figure 5.4 shows the Deformed Mesh of the slope and the critical slip surface (Figure5.4) is represented by Shear shadings of incremental strains.

$$FS = \frac{\text{available shear strength}}{\text{shear strength at failure}}$$

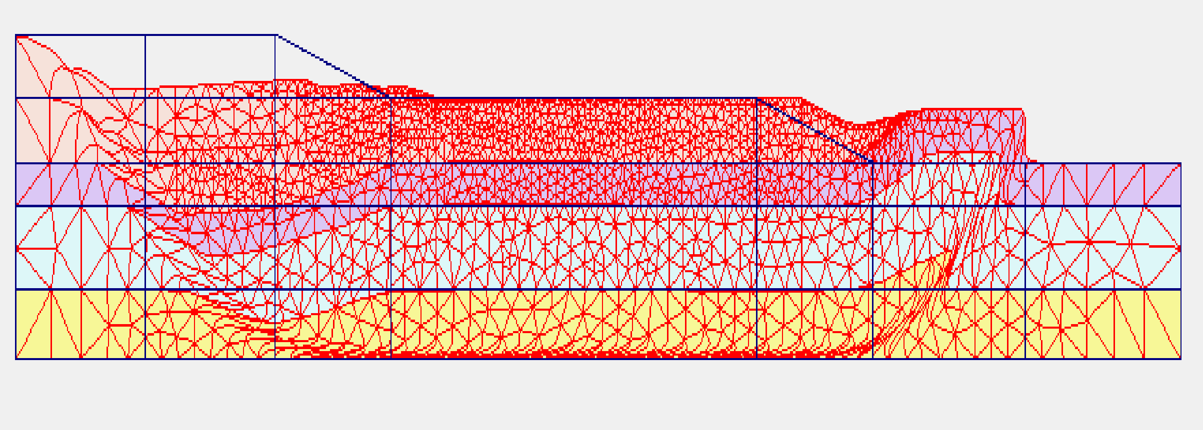


Figure 5.4: Deformed mesh of James bay case

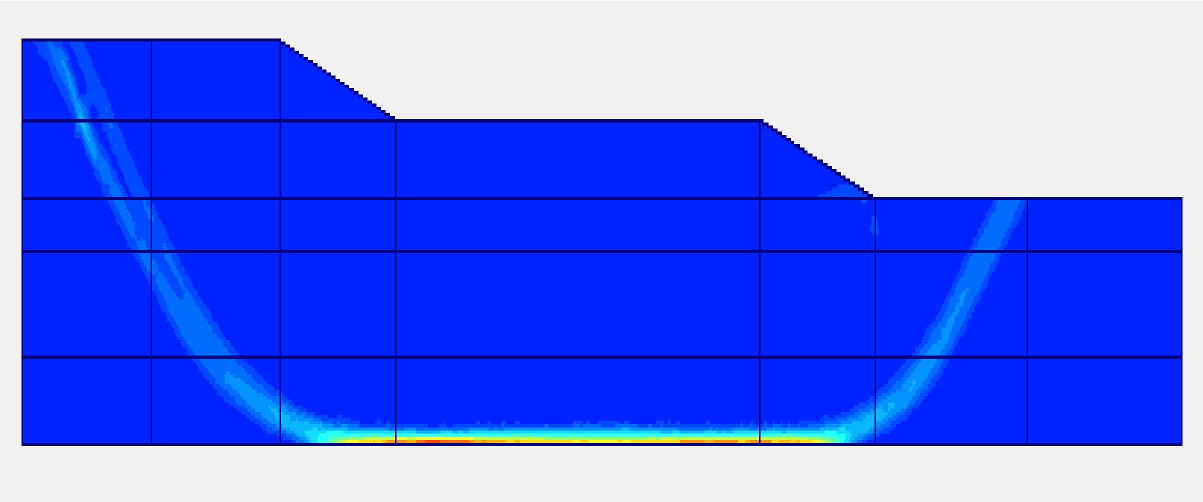


Figure 5.5: Shear shadings of incremental strains

5.1.2 Reliability-based Analysis

In this work, six input parameters that are considered variables are unit weight (γ) and friction angle of the embankment fill (ϕ), the thickness of the clay crust (t_{cr}), the undrained shear strength of marine (S_{uM}) and lacustrine clays (S_{uL}), and the depth of till layer (D_{till}). Input variables and their statistical parameters are shown in Table 5.2.

Table 5.2: Input variables and their statistical parameters

Input variable	Mean (μ)	Variance (Var)	Standard deviation (σ)
γ (kN/m^3)	20.0	1.00	1.00
ϕ (^{o})	30.0	1.00	1.00
t_{cr} (m)	4.0	0.23	0.48
S_{uM} (kPa)	34.5	66.26	8.14
S_{uL} (kPa)	31.2	74.82	8.65
D_{till} (m)	18.5	1.00	1.00

The parameters are assumed as uncorrelated normally distributed. For the six input parameters Full Factorial Design is used to generate $2^k = 2^6 = 64$ sets of data. The points in the experimental design for input parameters t_{cr} and D_{till} are estimated as ($\mu+3\sigma$) and ($\mu-3\sigma$) and for γ, ϕ, S_{uM}, S_{uL} as ($\mu+\sigma$) and ($\mu-\sigma$). The design has been done using Matlab (Math works 2010). FOS corresponding to 64 sets of data are analysed using PLAXIS and are tabulated (Table 5.3). The Mean μ[F], Variance V[F] and Standard Deviation σ[F] of Factors of safety are estimated using MS Excel.

Table 5.3: FOS of James Bay Dykes corresponding to 64 sample points using PLAXIS

	γ	ϕ	t_{cr}	S_{uM}	S_{uL}	D_{till}	FOS
Mean	20	30	4.00	34.5	32.4	18.5	1.242
Lower limit	21	31	5.44	42.64	39.85	21.5	-
Upper limit	19	29	2.56	26.36	22.55	15.5	-
1	21	31	5.44	42.64	39.85	21.5	1.336
2	21	31	5.44	42.64	39.85	15.5	1.496
3	21	31	5.44	42.64	22.55	21.5	0.960
4	21	31	5.44	42.64	22.55	15.5	1.117

5	21	31	5.44	26.36	39.85	21.5	1.234
6	21	31	5.44	26.36	39.85	15.5	1.225
7	21	31	5.44	26.36	22.55	21.5	0.867
8	21	31	5.44	26.36	22.55	15.5	1.024
9	21	31	2.56	42.64	39.85	21.5	1.341
10	21	31	2.56	42.64	39.85	15.5	1.501
11	21	31	2.56	42.64	22.55	21.5	0.961
12	21	31	2.56	42.64	22.55	15.5	1.119
13	21	31	2.56	26.36	39.85	21.5	1.188
14	21	31	2.56	26.36	39.85	15.5	1.180
15	21	31	2.56	26.36	22.55	21.5	0.834
16	21	31	2.56	26.36	22.55	15.5	0.977
17	21	29	5.44	42.64	39.85	21.5	1.327
18	21	29	5.44	42.64	39.85	15.5	1.483
19	21	29	5.44	42.64	22.55	21.5	0.951
20	21	29	5.44	42.64	22.55	15.5	1.108
21	21	29	5.44	26.36	39.85	21.5	1.224
22	21	29	5.44	26.36	39.85	15.5	1.213
23	21	29	5.44	26.36	22.55	21.5	0.863
24	21	29	5.44	26.36	22.55	15.5	1.015
25	21	29	2.56	42.64	39.85	21.5	1.330
26	21	29	2.56	42.64	39.85	15.5	1.486
27	21	29	2.56	42.64	22.55	21.5	0.954
28	21	29	2.56	42.64	22.55	15.5	1.112
29	21	29	2.56	26.36	39.85	21.5	1.177
30	21	29	2.56	26.36	39.85	15.5	1.221
31	21	29	2.56	26.36	22.55	21.5	0.832
32	21	29	2.56	26.36	22.55	15.5	0.968
33	19	31	5.44	42.64	39.85	21.5	1.461
34	19	31	5.44	42.64	39.85	15.5	1.625
35	19	31	5.44	42.64	22.55	21.5	1.046
36	19	31	5.44	42.64	22.55	15.5	1.218

37	19	31	5.44	26.36	39.85	21.5	1.353
38	19	31	5.44	26.36	39.85	15.5	1.332
39	19	31	5.44	26.36	22.55	21.5	0.967
40	19	31	5.44	26.36	22.55	15.5	1.113
41	19	31	2.56	42.64	39.85	21.5	1.464
42	19	31	2.56	42.64	39.85	15.5	1.629
43	19	31	2.56	42.64	22.55	21.5	1.050
44	19	31	2.56	42.64	22.55	15.5	1.222
45	19	31	2.56	26.36	39.85	21.5	1.736
46	19	31	2.56	26.36	39.85	15.5	1.282
47	19	31	2.56	26.36	22.55	21.5	0.929
48	19	31	2.56	26.36	22.55	15.5	1.072
49	19	29	5.44	42.64	39.85	21.5	1.451
50	19	29	5.44	42.64	39.85	15.5	1.541
51	19	29	5.44	42.64	22.55	21.5	1.040
52	19	29	5.44	42.64	22.55	15.5	1.208
53	19	29	5.44	26.36	39.85	21.5	1.331
54	19	29	5.44	26.36	39.85	15.5	1.319
55	19	29	5.44	26.36	22.55	21.5	0.962
56	19	29	5.44	26.36	22.55	15.5	1.104
57	19	29	2.56	42.64	39.85	21.5	1.455
58	19	29	2.56	42.64	39.85	15.5	1.544
59	19	29	2.56	42.64	22.55	21.5	1.044
60	19	29	2.56	42.64	22.55	15.5	1.212
61	19	29	2.56	26.36	39.85	21.5	1.416
62	19	29	2.56	26.36	39.85	15.5	1.270
63	19	29	2.56	26.36	22.55	21.5	0.925
64	19	29	2.56	26.36	22.55	15.5	1.063
						μ[F]=	1.203
						σ[F]=	0.218
						V[F]=	0.047

Using the FOS values obtained, regression analysis is done for the response surface model,

$$FOS = 1.39086 + (-0.05875 * \gamma) + (0.01109 * \phi) + (0.00022 * t_{cr}) + (0.00878 * S_{uM})$$

$$+ (0.02047 * S_{uL}) + (-0.01557 * D_{till})$$

($R^2 = 0.89$, $R^2_{adj} = 0.879$)

Solving for the reliability index using solver optimization tool, the value obtained is:

$\beta = 1.029$

The probability of failure $P_f = \Phi(-1.029) = 0.1517 = 15.17$ %

5.2 Limit Analysis Method using LimitState:GEO

5.2.1 Deterministic analysis of James Bay dykes

The slope is modelled using LimitState:GEO 3.2.d as shown in Figure 5.6. Mohr-coulomb model is considered to represent the soil. Nodal density is taken as very fine (2000 nodes). The soil parameters and the dimensions are taken from Table5.1 and Figure 5.1 respectively.

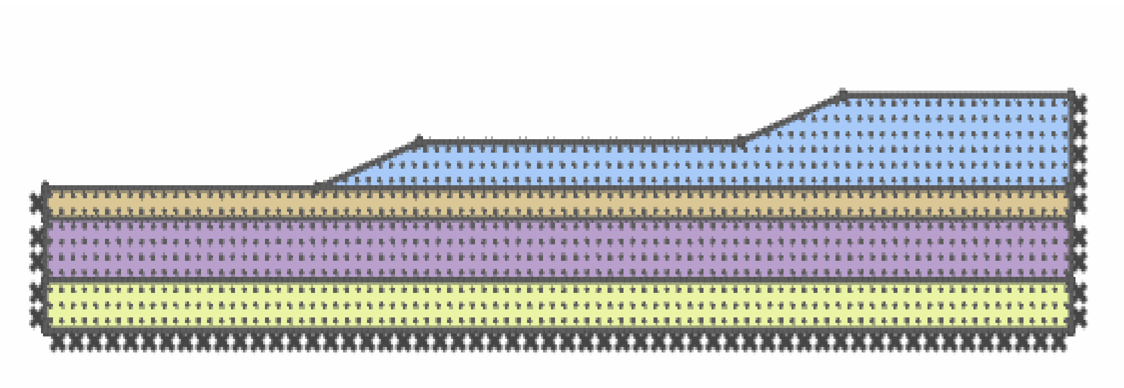


Figure 5.6: LimitState:GEO model of James Bay dykes

Factor of safety is indicated in terms of Adequacy factor for the Factor Strength. The failure mechanism and deformation of the slope is shown in Figure 5.7 and Figure 5.8 respectively.

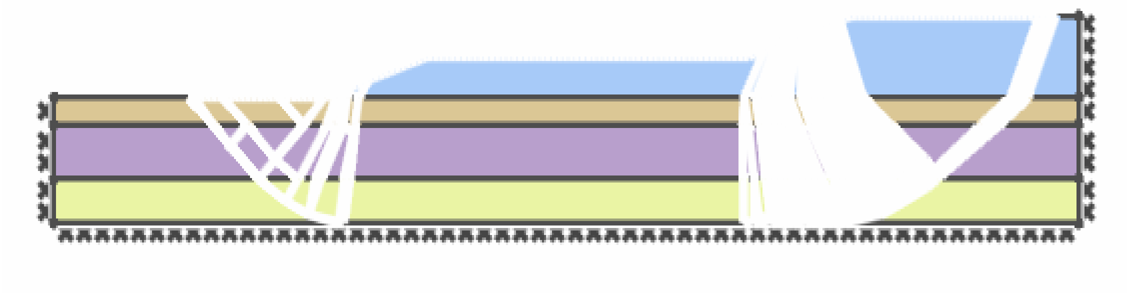


Figure5.7: Failure Mechanism in James Bay dykes

Figure 5.8: Deformed James Bay dykes

5.2.2 Reliability-based analysis of James Bay dykes

The FOS corresponding to the 64 sample points using LimitState:GEO along with its mean value μ[F], Standard deviation σ[F] and variance Var[F]are shown in Table 5.4.

Table 5.4: FOS of James Bay Dykes corr. to 64 sample points using LimitState:GEO

	γ	ϕ	t_{cr}	S_{uM}	S_{uL}	D_{till}	FOS
Mean	20	30	4.00	34.5	32.4	18.5	1.263
Lower limit	21	31	5.44	42.64	39.85	21.5	-
Upper limit	19	29	2.56	26.36	22.55	15.5	-
1	21	31	5.44	42.64	39.85	21.5	1.401
2	21	31	5.44	42.64	39.85	15.5	1.611

3	21	31	5.44	42.64	22.55	21.5	0.965
4	21	31	5.44	42.64	22.55	15.5	1.138
5	21	31	5.44	26.36	39.85	21.5	1.288
6	21	31	5.44	26.36	39.85	15.5	1.288
7	21	31	5.44	26.36	22.55	21.5	0.881
8	21	31	5.44	26.36	22.55	15.5	1.03
9	21	31	2.56	42.64	39.85	21.5	1.405
10	21	31	2.56	42.64	39.85	15.5	1.617
11	21	31	2.56	42.64	22.55	21.5	0.967
12	21	31	2.56	42.64	22.55	15.5	1.144
13	21	31	2.56	26.36	39.85	21.5	1.233
14	21	31	2.56	26.36	39.85	15.5	1.233
15	21	31	2.56	26.36	22.55	21.5	0.848
16	21	31	2.56	26.36	22.55	15.5	0.987
17	21	29	5.44	42.64	39.85	21.5	1.387
18	21	29	5.44	42.64	39.85	15.5	1.592
19	21	29	5.44	42.64	22.55	21.5	0.954
20	21	29	5.44	42.64	22.55	15.5	1.125
21	21	29	5.44	26.36	39.85	21.5	1.27
22	21	29	5.44	26.36	39.85	15.5	1.269
23	21	29	5.44	26.36	22.55	21.5	0.873
24	21	29	5.44	26.36	22.55	15.5	1.018
25	21	29	2.56	42.64	39.85	21.5	1.391

26	21	29	2.56	42.64	39.85	15.5	1.598
27	21	29	2.56	42.64	22.55	21.5	0.958
28	21	29	2.56	42.64	22.55	15.5	1.131
29	21	29	2.56	26.36	39.85	21.5	1.215
30	21	29	2.56	26.36	39.85	15.5	1.215
31	21	29	2.56	26.36	22.55	21.5	0.840
32	21	29	2.56	26.36	22.55	15.5	0.975
33	19	31	5.44	42.64	39.85	21.5	1.548
34	19	31	5.44	42.64	39.85	15.5	1.781
35	19	31	5.44	42.64	22.55	21.5	1.065
36	19	31	5.44	42.64	22.55	15.5	1.258
37	19	31	5.44	26.36	39.85	21.5	1.424
38	19	31	5.44	26.36	39.85	15.5	1.423
39	19	31	5.44	26.36	22.55	21.5	0.974
40	19	31	5.44	26.36	22.55	15.5	1.138
41	19	31	2.56	42.64	39.85	21.5	1.553
42	19	31	2.56	42.64	39.85	15.5	1.787
43	19	31	2.56	42.64	22.55	21.5	1.069
44	19	31	2.56	42.64	22.55	15.5	1.264
45	19	31	2.56	26.36	39.85	21.5	1.363
46	19	31	2.56	26.36	39.85	15.5	1.362
47	19	31	2.56	26.36	22.55	21.5	0.938
48	19	31	2.56	26.36	22.55	15.5	1.091

49	19	29	5.44	42.64	39.85	21.5	1.534
50	19	29	5.44	42.64	39.85	15.5	1.759
51	19	29	5.44	42.64	22.55	21.5	1.055
52	19	29	5.44	42.64	22.55	15.5	1.243
53	19	29	5.44	26.36	39.85	21.5	1.403
54	19	29	5.44	26.36	39.85	15.5	1.403
55	19	29	5.44	26.36	22.55	21.5	0.965
56	19	29	5.44	26.36	22.55	15.5	1.125
57	19	29	2.56	42.64	39.85	21.5	1.538
58	19	29	2.56	42.64	39.85	15.5	1.766
59	19	29	2.56	42.64	22.55	21.5	1.059
60	19	29	2.56	42.64	22.55	15.5	1.25
61	19	29	2.56	26.36	39.85	21.5	1.476
62	19	29	2.56	26.36	39.85	15.5	1.343
63	19	29	2.56	26.36	22.55	21.5	0.929
64	19	29	2.56	26.36	22.55	15.5	1.078
						μ[F]=	1.247
						σ[F]=	0.252
						V[F]=	0.063

Using the FOS values obtained, regression analysis is done for the response surface model,

$$FOS = 1.62414 + (-0.06430 * \gamma) + (0.00524 * \phi) + (0.00613 * t_{cr}) + (0.01155 * S_{uM}) + (0.02374 * S_{uL}) + (-0.02226 * D_{till})$$

($R^2 = 0.943$, $R^2_{adj} = 0.937$)

Solving for the reliability index using solver optimization tool, the value obtained is:

$\beta = 1.048$

The probability of failure $P_f = \Phi(-1.048) = 0.1474 = 14.74\%$

5.3 Analytical method for James Bay dykes

The analytical method given by Low, 1989 is applicable for single slope. So, James Bay dyke is modified into a single slope connecting the starting point of higher slope with end point of lower slope. The new geometry of James Bay dykes is shown in Figure 5.9.

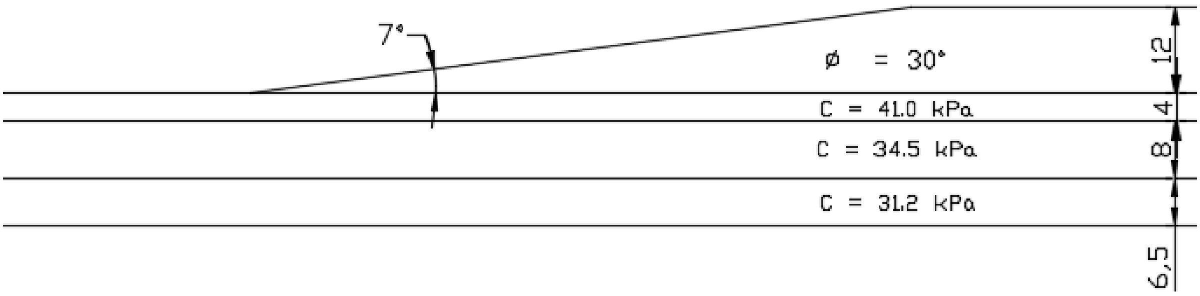


Figure 5.9: Geometry of James Bay dykes for Analytical method

5.3.1 Deterministic Analysis of James Bay dykes

For the slope in present study, the values assigned to each parameter in the equation are as follows:

$\beta = 7^{o}$; $C_m = 0$ kPa ; $\phi_m = 30^{o}$; H = 12m ; D = 18.5m ; $C_A = 35.17$ kPa

As the foundation soil is layered, C_A is calculated using weighted average of undrained shear stress of all the layers. Substituting the above values in the FOS equation, the obtained FOS is 1.143 with critical slip surface located at (x, y) = (46.2, 77.85). The critical slip circle of the slope is shown in Figure 5.10.

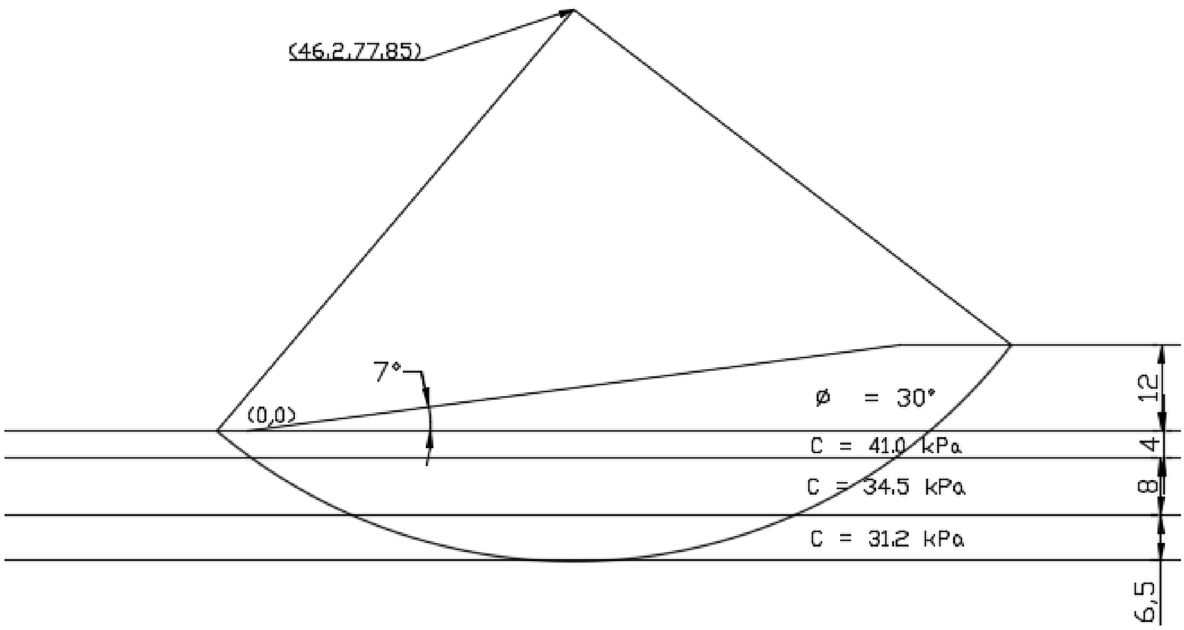


Figure 5.10: Critical slip circle for James Bay dykes using analytical method

5.3.2 Reliability-based Analysis of James Bay dykes

The FOS along with stability numbers N_1 and N_2 corresponding to 64 sampling points are presented in Table 5.5 using the analytical procedure.

Table 5.5: FOS of James Bay Dykes corr. to 64 sample points using Analytical Procedure

	γ	ϕ	t_{cr}	S_{uM}	S_{uL}	D_{till}	C_a	N_1	N_2	F_s
μ	20	30	4	34.5	32.4	18.5	35.17	6.676	1.013	1.143
max	21	31	5.44	42.64	39.85	21.5	-	-	-	-
min	19	29	2.56	26.36	22.55	15.5	-	-	-	-
1	21	31	5.44	42.64	39.85	21.5	40.99	6.455	0.855	1.236
2	21	31	5.44	42.64	39.85	15.5	41.43	6.961	1.236	1.416
3	21	31	5.44	42.64	22.55	21.5	33.35	6.455	0.855	1.030
4	21	31	5.44	42.64	22.55	15.5	37.53	6.961	1.236	1.303
5	21	31	5.44	26.36	39.85	21.5	36.03	6.455	0.855	1.102
6	21	31	5.44	26.36	39.85	15.5	34.54	6.961	1.236	1.216
7	21	31	5.44	26.36	22.55	21.5	28.38	6.455	0.855	0.896

8	21	31	5.44	26.36	22.55	15.5	30.64	6.961	1.236	1.103
9	21	31	2.56	42.64	39.85	21.5	41.21	6.455	0.855	1.241
10	21	31	2.56	42.64	39.85	15.5	41.74	6.961	1.236	1.425
11	21	31	2.56	42.64	22.55	21.5	33.57	6.455	0.855	1.036
12	21	31	2.56	42.64	22.55	15.5	37.83	6.961	1.236	1.312
13	21	31	2.56	26.36	39.85	21.5	34.06	6.455	0.855	1.049
14	21	31	2.56	26.36	39.85	15.5	31.82	6.961	1.236	1.137
15	21	31	2.56	26.36	22.55	21.5	26.42	6.455	0.855	0.844
16	21	31	2.56	26.36	22.55	15.5	27.92	6.961	1.236	1.024
17	21	29	5.44	42.64	39.85	21.5	40.99	6.455	0.855	1.236
18	21	29	5.44	42.64	39.85	15.5	41.43	6.961	1.236	1.416
19	21	29	5.44	42.64	22.55	21.5	33.35	6.455	0.855	1.030
20	21	29	5.44	42.64	22.55	15.5	37.53	6.961	1.236	1.303
21	21	29	5.44	26.36	39.85	21.5	36.03	6.455	0.855	1.102
22	21	29	5.44	26.36	39.85	15.5	34.54	6.961	1.236	1.216
23	21	29	5.44	26.36	22.55	21.5	28.38	6.455	0.855	0.896
24	21	29	5.44	26.36	22.55	15.5	30.64	6.961	1.236	1.103
25	21	29	2.56	42.64	39.85	21.5	41.21	6.455	0.855	1.241
26	21	29	2.56	42.64	39.85	15.5	41.74	6.961	1.236	1.425
27	21	29	2.56	42.64	22.55	21.5	33.57	6.455	0.855	1.036
28	21	29	2.56	42.64	22.55	15.5	37.83	6.961	1.236	1.312
29	21	29	2.56	26.36	39.85	21.5	34.06	6.455	0.855	1.049
30	21	29	2.56	26.36	39.85	15.5	31.82	6.961	1.236	1.137
31	21	29	2.56	26.36	22.55	21.5	26.42	6.455	0.855	0.844
32	21	29	2.56	26.36	22.55	15.5	27.92	6.961	1.236	1.024
33	19	31	5.44	42.64	39.85	21.5	40.99	6.455	0.855	1.236
34	19	31	5.44	42.64	39.85	15.5	41.43	6.961	1.236	1.416
35	19	31	5.44	42.64	22.55	21.5	33.35	6.455	0.855	1.030
36	19	31	5.44	42.64	22.55	15.5	37.53	6.961	1.236	1.303
37	19	31	5.44	26.36	39.85	21.5	36.03	6.455	0.855	1.102
38	19	31	5.44	26.36	39.85	15.5	34.54	6.961	1.236	1.216
39	19	31	5.44	26.36	22.55	21.5	28.38	6.455	0.855	0.896

40	19	31	5.44	26.36	22.55	15.5	30.64	6.961	1.236	1.103
41	19	31	2.56	42.64	39.85	21.5	41.21	6.455	0.855	1.241
42	19	31	2.56	42.64	39.85	15.5	41.74	6.961	1.236	1.425
43	19	31	2.56	42.64	22.55	21.5	33.57	6.455	0.855	1.036
44	19	31	2.56	42.64	22.55	15.5	37.83	6.961	1.236	1.312
45	19	31	2.56	26.36	39.85	21.5	34.06	6.455	0.855	1.049
46	19	31	2.56	26.36	39.85	15.5	31.82	6.961	1.236	1.137
47	19	31	2.56	26.36	22.55	21.5	26.42	6.455	0.855	0.844
48	19	31	2.56	26.36	22.55	15.5	27.92	6.961	1.236	1.024
49	19	29	5.44	42.64	39.85	21.5	40.99	6.455	0.855	1.236
50	19	29	5.44	42.64	39.85	15.5	41.43	6.961	1.236	1.416
51	19	29	5.44	42.64	22.55	21.5	33.35	6.455	0.855	1.030
52	19	29	5.44	42.64	22.55	15.5	37.53	6.961	1.236	1.303
53	19	29	5.44	26.36	39.85	21.5	36.03	6.455	0.855	1.102
54	19	29	5.44	26.36	39.85	15.5	34.54	6.961	1.236	1.216
55	19	29	5.44	26.36	22.55	21.5	28.38	6.455	0.855	0.896
56	19	29	5.44	26.36	22.55	15.5	30.64	6.961	1.236	1.103
57	19	29	2.56	42.64	39.85	21.5	41.21	6.455	0.855	1.241
58	19	29	2.56	42.64	39.85	15.5	41.74	6.961	1.236	1.425
59	19	29	2.56	42.64	22.55	21.5	33.57	6.455	0.855	1.036
60	19	29	2.56	42.64	22.55	15.5	37.83	6.961	1.236	1.312
61	19	29	2.56	26.36	39.85	21.5	34.06	6.455	0.855	1.049
62	19	29	2.56	26.36	39.85	15.5	31.82	6.961	1.236	1.137
63	19	29	2.56	26.36	22.55	21.5	26.42	6.455	0.855	0.844
64	19	29	2.56	26.36	22.55	15.5	27.92	6.961	1.236	1.024
									μ[F]=	1.148
									σ[F]=	0.166
									V[F]=	0.027

Using the FOS values obtained, regression analysis is done for the response surface model,

$$FOS = 0.96777 + (2.29038E\text{-}18 * \gamma) + (-1.90712E\text{-}34 * \phi) + (0.01015 * t_{cr}) + (0.01249 * S_{uM}) + (0.00921 * S_{uL}) + (-0.03127 * D_{till})$$

($R^2 = 0.952$, $R^2_{adj} = 0.947$)

Solving for the reliability index using solver optimization tool, the value obtained is:

$\beta = 1.114$

The probability of failure $P_f = \Phi(-1.114) = 0.1326 = 13.26$ %

The FOS, reliability index (β) and probability of failure (P_f) values obtained from above three analyses and from literature are tabulated (Table 5.6) as follows:

Table 5.6 Comparison of the outputs of different analysis approaches

Method of Analysis	$\mu(F)$	β	P_f
Spread sheet- based probabilistic slope analysis[a]	1.46	2.32	-
FOSM[a]	1.46	2.42	-
Simplified analysis[a]	1.46	1.84	-
Finite Element Method	1.203	1.029	
Limit Analysis Method	1.247	1.048	14.74
Analytical Method	1.148	1.114	13.26

[a] the values are obtained based on analysis done by Ramly et al. (2002)

CHAPTER 6

Conclusion

In case of a hypothetical slope, the reliability index obtained from Finite Element Method (1.69) is significantly lower than that found from Analytical approach (2.05). This is because the analytical approach considers the embankment on soft soil and the shape and location of critical slip surface is assumed beforehand. As the limit analysis method (1.94) considers the rigid soil movement, the reliability index is higher than that from the finite element method.

In case of James Bay dykes, it is found that the reliability index (1.029) calculated by finite element analysis based on the response surface approximation of the finite-element method is significantly lower than that (2.32) reported by Ramly et al. (2002), based on Bishop's simplified method. This is because the finite element method predicts the critical failure mechanisms better than the Bishop's simplified method. For multiple soil layers, the factor of safety evaluated by the finite-element method using strength reduction technique depends on, to a certain extent, the values of other input parameters such as Young's moduli, Poisson's ratios, and modulus/strength ratios.

Based on the results of the analyses presented, it can be seen that the deterministic stability analysis model can significantly affect the results of reliability analyses of embankments. Any simplification of the deterministic analytical model, such as a circular slip surface or adopting a nonrigorous method, can lead to an inaccurate estimation of the reliability index. In particular, when the factor of safety is sensitive to the assumed shape of the slip surface, the reliability index obtained by assuming circular slip surface analysis will be greatly overestimated. The advantages of the reliability analysis method based on the response surface method are its ability to integrate the deterministic methods and FORM into a probabilistic stability analysis.

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